(b) An experiment was performed to investigate how the behavior of mozzarella cheese varied with temperature. The following observations on x = Temperature and y = elongation(%) at failure of the cheese.

X	59	63	68	72	74	78	83
y	118	182	247	208	197	135	132

(i) Determine the equation of the estimated regression line using the principle of least square.

(ii) Estimate the elongation at failure of the cheese when the temperature is 70.

(c) The inside diameter of a randomly selected piston ring is a random variable with mean value of 12 cm and standard deviation 0.04 cm. If $\overline{X}$ is the sample mean diameter for a random sample of n = 16 rings,

(i) where is the sampling distribution of $\overline{X}$ centered,

(ii) what is the standard deviation of the $\overline{X}$ distribution.

(iii) How likely is it that the sample mean diameter exceeds 12.01?

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1629   G

Unique Paper Code   : 2352011103

Name of the Paper   : DSC-3: Probability and Statistics

Name of the Course   : **B.Sc. (H) Mathematics**
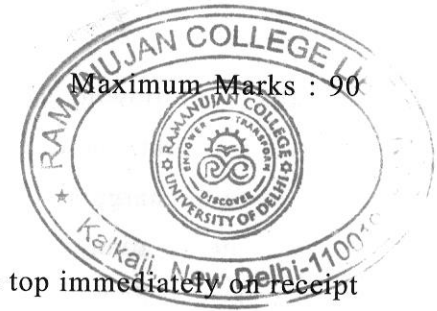
Semester   : I

Duration : 3 Hours   Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory.

3. Attempt any **two** parts from each question.

4. **All** questions carry equal marks.

5. Use of non-programmable scientific calculators and statistical tables is permitted.

P.T.O.

1. (a) The following table gives the accompanying specific gravity values for various wood types used in construction. Construct a stem and leaf display and comment on any interesting features of the display

.31	.35	.36	.36	.37	.38	.40	.40	.40
.41	.41	.42	.42	.42	.42	.42	.43	.44
.45	.46	.46	.47	.48	.48	.48	.51	.54
.54	.55	.58	.62	.66	.66	.67	.68	.75

(b) The following data consists of observations on the time until failure (1000s of hours) for a sample of turbochargers from one type of engine. Compute the Median, Upper Fourth (third quartile) and Lower Fourth (first quartile)

1.6	2.0	2.6	3.0	3.9	3.5	4.5	4.6	4.8	5.0

(c) The following table gives the data on oxidation-induction time (measured in minutes) for various commercial oils.

87	103	130	160	180	195	132	145	211	105

(i) Calculate the sample variance and standard deviation.

$$F(x) = \begin{cases} 0 & x < 0 \\ \dfrac{x^2}{4} & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

(i) Calculate $P(.5 \le X \le 1)$.

(ii) What is the median checkout duration $\tilde{\mu}$?

(iii) Obtain the density function f(x).

(c) The amount of distilled water dispensed by a certain machine is normally distributed with mean value 64 oz and standard deviation 0.78 oz. What container size c will ensure that overflow occurs only 0.5% of the time?

6. (a) Toughness and fibrousness of asparagus are major determinants of quality. This was reported in a study with the following data on x = shear force (kg) and y = percent fiber dry weight.

X	46	48	55	57	60	72	81	85	94	109
y	2.18	2.10	2.13	2.28	2.34	2.53	2.28	2.62	2.63	2.50

(i) Calculate the value of the sample correlation coefficient. Based on this value, how would you describe the nature of relationship between the two variables?

(ii) If shear force is expressed in pounds, what happens to the value of r? Why?

P.T.O.

What is the probability that the reaction time is between 1.00 sec and 1.75 sec? If 2 sec is a critical long reaction time, what is the probability that actual reaction time will exceed this value?

(c) If X is a binomially distributed random variable with parameters n and p, prove that

(i) $E[X] = np$

(ii) $V[X] = np(1 - p)$

5. (a) If 75% of all purchases in a certain store are made with a credit card and the random variable, X = number among ten randomly selected purchases made with a credit card is a Binomial variate, then determine

(i) $E(X)$

(ii) $V(X)$

(iii) σ_X

(iv) The probability that X is within 1 standard deviation of its mean value.

(b) Let X denote the amount of time a book on two-hour reserve is actually checked out, and suppose the cumulative density function is

(ii) If the observations were re-expressed in hours, what would be the resulting values of the sample variance and sample standard deviation? Answer without reperforming the calculations.

2. (a) If A and B are any two events, then show that $P(A \cap B') = P(A) - P(A \cap B)$. Hence or otherwise prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

(b) Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft not discovered do not have such a locator. Suppose a light aircraft has disappeared. If it has an emergency locator, what is the probability that it will not be discovered?

(c) State Baye's Theorem A large operator of timeshare complexes requires anyone interested in making a purchase to first visit the site of interest. Historical data indicates that 20% of all potential purchasers select a day visit, 50% choose a one-night visit, and 30% opt for a two- night visit. In addition, 10% of day visitors ultimately

make a purchase, 30% of one-night visitors buy a unit, and 20% of those visiting for two nights decide to buy. Suppose a visitor is randomly selected and is found to have made a purchase. How likely is it that this person made a day visit?

3. (a) In a group of five potential blood donors a, b, c, d, and e, only a and b have Opositive (O+) blood type. Five blood samples, one from each individual, will be typed in random order until an O+ individual is identified. Let the random variable Y = the number of typings necessary to identify an O+ individual.

   (i) Find the probability mass function (pmf) of Y.

   (ii) Draw the line graph and probability histogram of the pmf.

(b) The n candidates for a job have been ranked 1,2,3, ... n. Each candidate has an equal chance of being selected for the job. Let the random variable X be defined as

X = the rank of a randomly selected candidate

   (i) Find the probability mass function (pmf) of X.

   (ii) Compute E(X) and V(X).

(c) For any random variable X, prove that $V(aX + b) = a^2 V(X)$ and $\sigma_{aX+b} = |a|\sigma_X$.

4. (a) The distribution of the amount of gravel (in tons) sold by a particular construction supply company in a given week is a continuous random variable X with probability density function (pdf)

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

   (i) the cumulative density function (cdf) of sales

   (ii) E(X)

   (iii) V(X)

   (iv) σ_X

(b) The reaction time for an in-traffic response to a brake signal from standard brake lights can be modelled with a normal distribution having mean value 1.25 sec and standard deviation of 0.46 sec.

P.T.O.