(c) A large tank contains 100 litres of salt water. Initially s_0 kg of salt is dissolved. Salt water flows into the tank at the rate of 10 litres per minute, and the concentration $c_{in}(t)$ (kg of salt/litre) of this incoming water-salt mixture varies with time. We assume that the solution in the tank is thoroughly mixed and that the salt solution flows out at the same rate at which it flows in: that is, the volume of water-salt mixture in the tank remains constant. Find a differential equation for the amount of salt in the tank at any time t.

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 4159 **H**

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : **B.Sc. (Hons.) Mathematics – DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts of each question.
3. Each part carries **7.5** marks.
4. Use of non-programmable Scientific Calculator is allowed.

1. (a) Solve

$$\{y^2(x + 1) + y\}dx + (2xy + 1)dy = 0$$

(b) Solve the initial value problem

$$x\frac{dy}{dx}\,y = y^2 \log x,\ y(1) = 2$$

(c) (i) Solve

$$(y \sec^2 x + \sec x \tan x)dx + (\tan x + 2y)dy = 0$$

(ii) Solve by reducing the order $y'' = (x + y')^2$

2. (a) Suppose that a mineral body formed in an ancient cataclysm originally contained the uranium isotope ^{238}U, (which has a half-life of 4.51×10^9 years) but no lead, the end product of the radioactive decay of ^{238}U. If today the ratio of ^{238}U atoms to lead atoms in the mineral body is 0.9, when did the cataclysm occur?

(b) Upon the birth of their first child, a couple deposited Rs. 10,000 in an account that pays 8% interest compounded continuously. The

6. (a) The Earth's atmospheric pressure p is often modelled by assuming that $\frac{dp}{dx}$ (the rate at which pressure p changes with altitude h above sea level) is proportional to p. Suppose that the pressure at sea level is 1013 millibars and that the pressure at an altitude of 20 km is 50 millibars. Use an exponential decay model

$$\frac{dp}{dx} = kp$$

to describe the system, and then by solving the equation, find an expression for p in terms of h. Determine k and the constant of integration from the initial conditions. What is the atmospheric pressure at an altitude of 50 km?

(b) Discuss Phase Plane Analysis of predator-prey model.

predators that compete for a single prey food-source is

$$\frac{dX}{dt} = a_1X - b_1XY - c_1XZ, \frac{dY}{dt} = a_2XY - b_2Y, \frac{dZ}{dt} = a_3XZ - b_3Z$$

where a_i, b_i, c_i for i = 1,2,3 are all positive constants. Here X(t) is the prey density and Y(t), Z(t) are the two predator densities. Find all possible equilibrium populations.

(c) Suppose a population can be modeled using the differential equation

$$\frac{dX}{dt} = 0.2X - 0.001X^2$$

with an initial population size of x_0 = 100 and a time step of 1 month. Find the predicted population after 2 months.

interest payment is allowed to accumulate. In how many years will the amount double? How much will the account contain on the child's 18th birthday?

(c) A roast initially at 50°F, is placed in a 375°F oven at 5 pm. After 75 minutes, it is found that the temperature of the roast is 125°F. When will the roast be 150°F?

3. (a) Show that the solutions e^x, e^{-x}, e^{-2x} of the third order differential equation

$$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$$

are linearly independent. Find the particular solution satisfying the given initial condition.

$$y(0) = 1,\ y'(0) = 2,\ y''(0) = 0$$

(b) Solve the differential equation using the method of Variation of Parameters

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 2e^{2x}$$

(c) Find the general solution of the differential equation using the method of Undetermined Coefficients.

$$\frac{d^2y}{dx^2} + 9y = 2\cos 3x$$

4. (a) Use the operator method to find the general solution of the following linear system

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t$$

$$\frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$

(b) Find the general solution of the differential equation. Assume $x > 0$.

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 72x^5$$

(c) A body with mass $m = \frac{1}{5}$ kg is attached to the end of a spring that is stretched 4m by a force of 20N. It is set in motion with initial position $x_0 = 1$m and initial velocity $v_0 = -5$m/s. Find the position function of the body as well as the amplitude, frequency and period of oscillation.

5. (a) Develop a model with two differential equations describing a predator-prey interaction.

(b) Define equilibrium solution of differential equation. A model of a three species interaction with two