(b) Convert the following Game problem into a linear programming problem for Player A and Player B and solve it by Simplex method.

	Player B		
Player A	3	-2	4
	-1	4	2

(c) Using Simplex method, solve the system of equations :

$$3x_1 + x_2 = 7$$

$$x_1 + x_2 = 3$$

Also write the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$.

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 3183 **H**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF) – B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each questions.
3. **All** questions carry equal marks.

1. (a) Define a Convex Set. Show that the set S defined as :

$$S = \{(x, y) \mid y^2 \leq 4x\} \text{ is a Convex Set.}$$

P.T.O.

(b) Let $x_1 = 1, x_2 = 2, x_3 = 4$ be a feasible solution to the system of equations :

$$2x_1 + 3x_2 - x_3 = 4$$

$$3x_1 - x_2 + x_3 = 5$$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Consider the following linear programming problem:

Minimize $z = cx$

subject to $Ax = b, x \geq 0$

Let $(x_B, 0)$ be a basic feasible solution with objective function value z_B corresponding to a basis B where $x_B = B^{-1}b$. By entering an a_j with $z_j - c_j > 0$ and removing a b_r subject to :

$$\frac{x_{Br}}{y_{rj}} = \text{Min}\left[\frac{x_{Bi}}{y_{ij}} : y_{ij} > 0\right]$$

Show that we can get a new feasible solution with improved value of the objective function compared to z_B.

(c) For the following cost minimization Transportation Problem, find initial basic feasible solution by using North-West comer rule, Least Cost method and Vogel's approximation method. Compare the three solutions (in terms of cost).

	A	B	C	D	Supply
I	19	14	23	11	11
II	15	16	12	21	13
III	30	25	16	39	19
Demand	6	10	12	15	

5. (a) Define the Saddle point. The pay-off matrix of a game is given below. Find the best strategy for each player, and the value of a play of the game of A and B.

Player B

Player A		I	II	III	IV	V
	I	9	3	1	8	0
	II	6	5	4	6	7
	III	2	4	3	3	8
	IV	5	6	2	2	1

	a	b	c	d
A	8	9	6	3
B	6	11	5	10
C	3	8	7	9

Find the optimal schedule and minimum total transport cost.

(b) A Company is faced with the problem of assigning six different machines to six different jobs. Determine the optimal solution of the Assignment Problem with the following cost matrix :

	a	b	c	d	e	f
1	9	22	58	11	19	27
2	43	78	72	50	63	48
3	41	28	91	37	45	33
4	74	42	27	49	39	32
5	36	11	57	22	25	18
6	3	56	53	31	17	28

2. (a) Using Simplex method, find the solution of the following linear programming problem :

Maximize $z = x_1 - 2x_2 + x_3$

subject to $x_1 + 2x_2 + x_3 \leq 12$

$2x_1 + x_2 - x_3 \leq 6$

$x_1 - 3x_2 \geq -9$

$x_1, x_2, x_3 \geq 0.$

(b) Using two phase method, solve the linear programming problem :

Minimize $z = -3x_1 + x_2$

subject to $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0.$

(c) Solve the following linear programming problem by Big - M method :

Maximize $z = 3x_1 + 2x_2 + 3x_3$

subject to $2x_1 + x_2 + x_3 \leq 2$

$3x_1 + 4x_2 + 2x_3 \geq 8$

$x_1, x_2, x_3 \geq 0.$

3. (a) Consider the following primal problem (P) and dual problem (D) :

(P) Minimize $z = cx$

Subject to $Ax \geq b$, $x \geq 0$

(D) Maximize $z = wb$

Subject to $wA \leq c$, $w \geq 0$,

If x_0 (w_0) is an optimal solution to the primal (dual) problem then there exists a feasible solution $w_0(x_0)$ to the dual (primal) such that $cx_0 = w_0b$.

(b) Use graphical method to solve the dual of the following linear programming problem :

Minimize $z = 6x_1 + 8x_2 + 7x_3 + 15x_4$

Subject to $x_1 + x_3 + 3x_4 \geq 4$

$x_2 + x_3 + x_4 \geq 3$

$x_1, x_2, x_3, x_4 \geq 0$

Further, find an optimal solution to the given problem from optimal solution of the dual problem.

(c) Obtain the dual of the following linear programming problem :

Maximize $z = 10x_1 + x_2 + 2x_3$

Subject to $x_1 + x_2 - 2x_3 \geq 10$

$x_1 + 4x_2 - 3x_3 = 3$

$4x_1 + x_2 + x_3 \leq 20$

$x_1 \geq$, $x_2 \leq 0$, and x_3 unrestricted in sign.

4. (a) A Company has four warehouses, a, b, c and d. It is required to deliver a product from these warehouses to three customers A, B and C. The warehouses have the following amounts in stock.

Warehouse :	a	b	c	d
No. of units :	15	16	12	13

and the customer's requirements are

Customers :	A	B	C
No. of units	18	20	18

The table below shows the costs of transporting one unit from warehouses to the customers :