

(d) Check the following series for absolute or conditional convergence :

$$(i) \sum (-1)^{n+1} \left(\frac{n}{n(n+3)} \right)$$

$$(ii) \sum (-1)^{n+1} \left(\frac{1}{n+1} \right)$$

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1591

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Unique Paper Code : 2352011102

Name of the Paper : DSC-2 : Elementary Real Analysis

Name of the Course : B.Sc. (H) Mathematics (UGCF-2022)

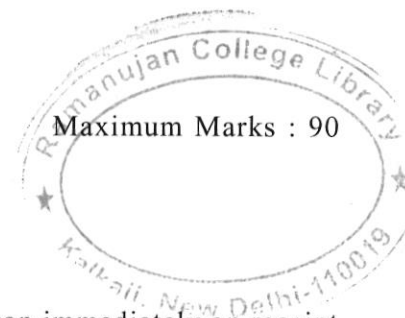
Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
 2. Attempt any **three** parts from each question.
 3. **All** questions carry equal marks.
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1. (a) Let $a \geq 0, b \geq 0$ prove that $a^2 \leq b^2 \Leftrightarrow a \leq b$.



(b) Determine and sketch the set of pairs (x, y) on $\mathbb{R} \times \mathbb{R}$ satisfying the inequality $|x| \leq |y|$.

(c) Find the supremum and infimum, if they exist, of the following sets :

$$(i) \left\{ \sin \frac{n\pi}{2} : n \in \mathbb{N} \right\}$$

$$(ii) \left\{ \left(\frac{1}{x} : x > 0 \right) \right\}$$

(d) Show that $\text{Sup} \left\{ 1 + \frac{1}{n} : n \in \mathbb{N} \right\} = 2$.

2. (a) Let S be a non-empty bounded subset of \mathbb{R} . Let $a > 0$ and let $aS = \{as : s \in S\}$. Prove that

$$\text{Sup} (aS) = a(\text{Sup } S)$$

6. (a) State the Root Test (limit form) for positive series. Using this test or otherwise, check the convergence of the following series

$$(i) \sum \left(n^{1/n} - 1 \right)^n$$

$$(ii) \sum \left(\frac{n^{n^2}}{(n+1)^{n^2}} \right)$$

(b) Check the convergence of the following series :

$$(i) \sum_{n=2}^{\infty} \left(\frac{1}{n \log n} \right)$$

$$(ii) \sum \left(\frac{n!}{n^n} \right)$$

(c) Define absolute convergence of a series. Show that every absolutely convergent series is convergent. Is the converse true? Justify your answer.

(b) Determine, if the following series converges, using

the definition of convergence, $\sum \log\left(\frac{a_n}{a_{n+1}}\right)$ given

that $a_n > 0$ for each n , $\lim_{n \rightarrow \infty} a_n = a$, $a > 0$.

(c) Find the rational number which is the sum of the series represented by the repeating decimal

$0.\overline{987}$.

(d) Check the convergence of the following series :

(i) $\sum \frac{1}{2^n + n}$

(ii) $\sum \sin\left(\frac{1}{n^2}\right)$

(b) If x and y are positive rational numbers with $x < y$, then show that there exists a rational number r such that $x < r < y$.

(c) Show that $\inf\left\{\frac{1}{n} : n \in \mathbb{N}\right\} = 0$.

(d) Show that every convergent sequence is bounded. Is the converse true? Justify.

3. (a) Using definition of limit, show that

$$\lim_{n \rightarrow \infty} \frac{n^2 + 3n + 5}{2n^2 + 5n + 7} = \frac{1}{2}$$

(b) Show that if $c > 0$, $\lim_{n \rightarrow \infty} (c)^{1/n} = 1$.

(c) Show that, if $x_n \geq 0$ for all n , and $\langle x_n \rangle$ is convergent

then $\langle \sqrt{x_n} \rangle$ is also convergent and

$$\lim_{n \rightarrow \infty} \sqrt{x_n} = \sqrt{\lim_{n \rightarrow \infty} x_n}$$

(d) Show that every increasing sequence which is bounded above is convergent.

4. (a) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2x_n}$ for all n . Prove that

$\langle x_n \rangle$ is convergent and find its limit.

(b) Prove that every Cauchy sequence is convergent.

(c) Show that the sequence $\langle x_n \rangle$ defined by

$$x_n = 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots + \frac{1}{n!}, \text{ for all } n \in \mathbb{N}$$

is convergent.

(d) Find the limit superior and limit inferior of the following sequences :

(i) $x_n = (-1)^n \left(1 + \frac{1}{n}\right)$, for all $n \in \mathbb{N}$

(ii) $x_n = \left(1 + \frac{1}{n}\right)^{n+1}$, for all $n \in \mathbb{N}$

5. (a) Show that if a series $\sum a_n$ converges, then the sequence $\langle a_n \rangle$ converges to 0.