distance $d_{JC}(S_0,S_1)$ to 4 decimal digits, from the following 40 base table :

$S_1\backslash S_0$	A	G	C	T
A	7	0	1	1
G	1	9	2	0
C	0	2	7	2
T	1	0	1	6

(c) Explain Kimura 2-parameter and 3-parameter models along with their corresponding distance formulas.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4226 **H**

Unique Paper Code : 2353012004

Name of the Paper : DSE: Bio-Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** question by selecting two parts from each question.

3. **All** questions carry equal marks.

4. Use of non-programmable Scientific Calculator allowed.

1. Attempt any **two** parts :

(a) Develop a continuous model for population growth using the given data, postulating a linear relationship between the population growth rate and the population size itself. Utilize this model to forecast the U.S. population in the year 3000.

Year	U.S. Population
1800	5.3
1810	7.2
1820	9.6
1830	12.9
1840	17.1
1850	23.2
1860	31.4

(b) Find equilibrium states within the context of the logistic growth model $\frac{dP}{dt} = \left(1 - \frac{P}{K}\right)P$. Identify the model's equilibrium state and classify them as stable or unstable.

(c) A drug is available in dose M mg, which raises the blood plasma concentration of an average adult by 10 mg l^{-1}. Once in the blood plasma, the drug concentration decays according to the equation

(b) What is Hopf bifurcation? Show that Hopf bifurcation holds for the following system

$$\dot{x} = -y + x(\mu - x^2 - y^2)$$
$$\dot{y} = x + y(\mu - x^2 - y^2)$$

(c) Show that the iteration scheme $x_{n+1} = 1 - \mu x_n(1 - x_n)$. Has a stable fixed point $x_0 = 1$ for $\mu < 4$ and that $\mu = 1$ is a bifurcation point where fixed point $x_0 = 1/\mu$ appears. Show that the periodic doubling occurs as soon as μ exceeds 3.

6. Attempt any **two** parts :

(a) Show that the following system has limit cycle.

$$\frac{du}{dt} = u(1-u)(u-a) - w + I(t),$$
$$\frac{dw}{dt} = bu - \gamma w, \quad 0 < a < 1,\ b > 0,\ \gamma \geq 0.$$

(b) Derive the formula for the Jukes-Cantor distance (d_{JC}) given that all the diagonal entries of Jukes-Cantor matrix M^t are $\frac{1}{4} + \frac{3}{4}\left(1 - \frac{4}{3}\alpha\right)^t$, where α is the mutation rate. Compute the Jukes- Cantor

(b) State Poincare-Bendixson theorem. Use theorem to show that the system

$$\frac{dx}{dt} = x - y - x^3$$
$$\frac{dy}{dt} = x + y - y^3$$

has a stable periodic orbit.

(c) Examine what fixed points arise in the phase plane for

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + ax = 0$$

where a and b are constants ($a \neq 0$).

5. Attempt any **two** parts :

(a) What is bifurcation? Explain the concept of bifurcation with the creation of equilibrium points for the following system

$$\dot{x} = y$$
$$\dot{y} = \mu - x^2 - \mu^{1/4}$$

$\frac{d}{dt}C(t) = \frac{-C(t)}{\tau}$ with $\tau = 4$ hours. Health regulation require that the concentration never exceed 15 mg l^{-1}. What is shortest safe time interval at which doses can be given regularly? What is the minimum concentration if doses are given at this time interval.

2. Attempt any **two** parts :

(a) When a drug is administered, it forms a concentration in the body fluids. This concentration diminishes in time through elimination, destruction or invaction. The rate of reduction of the concentration is proportional to the concentration. Prove that the maximum concentration is $C_M = \frac{C_0}{1 - e^{t_0/\tau}}$, where C_0 is the concentration of the drug at $t = 0$, τ is constant which measure the rapidity at which concentration of drug falls.

(b) Using the law of mass action, derive a mathematical model governed by the intermediaries X and Y in the trimolecular reaction :

$$\begin{aligned} A &\longrightarrow X \\ B + X &\longrightarrow Y + D \\ 2X + Y &\longrightarrow 3X \\ X &\longrightarrow E \end{aligned}$$

where A, B, D, and E are initial and final products, and all rate constants are equal to 1.

(c) The rate of increase of bacteria in a culture is proportional to the number present. The population multiplies by the factor n in the time interval t. The population is found to increase by 2455 bacteria from t = 2 to t = 3 and by 4314 bacteria from t = 4 to t = 5. If the initial population is p_0, then show that $p_0 = 4291$ approximately and that, when t = 3, n is about 2.33.

3. Attempt any **two** parts :

(a) Let $\bar{d}$ denote the average length of the infection and β be the per capita recovery rate in the SIS model. Give the mathematical justification for $\bar{d} = 1/\beta$.

(b) Write the SIR model for infectious diseases along with properly defining all the parameters used. Also mention the assumptions undertaken to derive the model. Give the schematic representation of the SIR model. Will S(t) be increasing or decreasing? Give two reasons: one based on physical considerations, the other on knowing the derivative.

(c) Show that if

$$\lim_{t\to\infty} \frac{dR}{dt} > 0, \quad \text{then} \quad \lim_{t\to\infty} R(t) \to \infty.$$

Deduce that in the SIR model, eventually the number of infectives goes to 0 i.e:, $I(\infty) = 0$.

4. Attempt any **two** parts :

(a) For the predator-prey model

$$\frac{dV}{dt} = \frac{2}{3}V - \frac{V^2}{6} - \frac{VO}{1+V}$$
$$\frac{dO}{dt} = \delta O\left(1 - \frac{O}{V}\right)$$

Find the null clines and the directions of movement along the null clines. Also represent the same graphically.