(c) Define independent set and edge cover of a graph with example. Compute the maximum size of independent set and minimum size of edge cover in C_5 and k_5 (where C_n is a n-cycle). (7.5)

9/1/24 (Mor)

[This question paper contains 12 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1694 G

Unique Paper Code : 2353012001

Name of the Paper : Graph Theory

Name of the Course : B. Sc. (Hons.) Mathematics-DSE

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

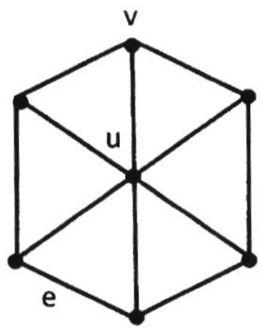
2. All questions has three parts **(a)**, **(b)** and **(c)**. You have to attempt any **two** parts of each question.

3. **All** questions carry equal marks.

4. Parts of each question to be attempted together.

5. Use of Calculator not allowed.

1. (a) (i) Define sub-graph of a graph. Draw pictures of the sub-graphs of $G\setminus\{e\}$, $G\setminus\{v\}$ and $G\setminus\{u\}$ of the following graph G.



(ii) Determine that if there exist a graph whose degree sequence is 5,4,4,3,2,1. Either draw a graph or explain why no such graph exists.

Draw a graph with degree sequence 4,3,2,2,1.

(4.5,3)

(b) Define planar and nonplanar graph with example. If G is a connected planar graph with e edges and n vertices, where $n \geq 3$, then prove that $e \leq 3n - 6$. Hence prove that k_5 is nonplanar. (7.5)

(c) State and explain Kuratowski's theorem. Let G be a connected plane graph with e edges and n vertices such that every region of G has at least five edges on its boundary, then show that $3e \leq 5n - 10$. (7.5)

6. (a) State and explain four color problem. Let $\Delta(G)$ be the maximum of the degrees of the vertices of a graph G, then show that $\chi(G) \leq 1 + \Delta(G)$.

(7.5)

(b) State and explain Hall's marriage theorem. Let G be a bipartite graph with bipartition sets V_1, V_2 in which every vertex has the same degree k. Show that G has a matching which saturates V_1.

(7.5)

What is the weight of the minimum tree and show your steps. (7.5)

(c) (i) Write the definition of a bridge for a graph G along with an example.

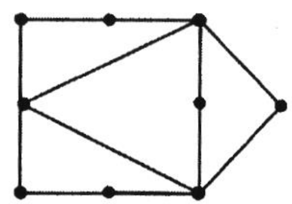
(ii) Let G be a connected graph of order n. If every edge of the graph is a bridge, then what is the total number of edges in the graph G. Give an example.

(iii) Give an example of a connected graph G with the properties that every bridge of G is adjacent to an edge that is not a bridge and every edge of G that is not a bridge is adjacent to a bridge. (2.5,2.5,2.5)

5. (a) Define vertex-connectivity and edge-connectivity of a graph. What is the relationship among vertex-connectivity, edge-connectivity and minimum degree of a graph? Verify it for k_4 and $k_{3,3}$. (7.5)

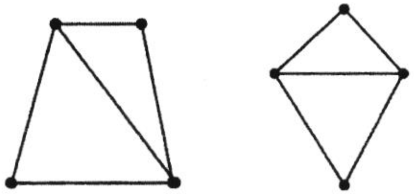
(b) (i) Define a complete graph. Does there exists a graph G with 30 edges and 10 vertices; each of degree 4 or 5. Justify your answer?

(ii) What is a bipartite graph? Determine whether the graph given below is bipartite. Give the bipartition sets or explain why the graph is not bipartite.

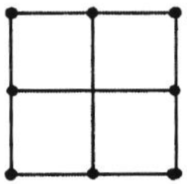


If bipartite then determine whether it is complete bipartite. (3,4.5)

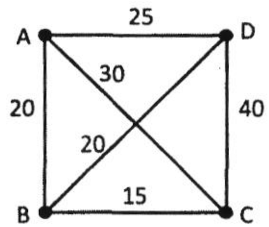
(c) (i) Define the term Isomorphic Graphs. For the below pair of graphs, either label the graphs so as to exhibit an isomorphism or explain why graphs are not isomorphic :



(ii) Solve the Chinese Postman Problem for the graph below :

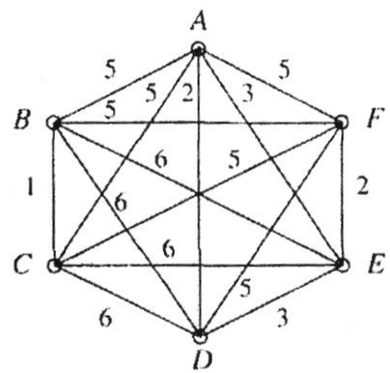


(4,3.5)

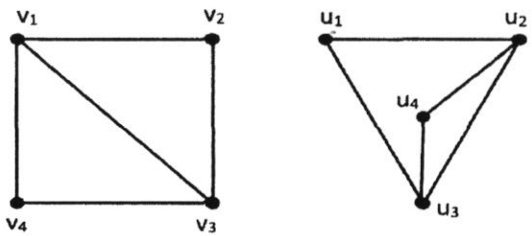


(4,3.5)

(b) What is the spanning tree and minimum spanning tree for a connected graph G? Use Kruskal's algorithm to find a spanning tree of minimum total weight for the following graph

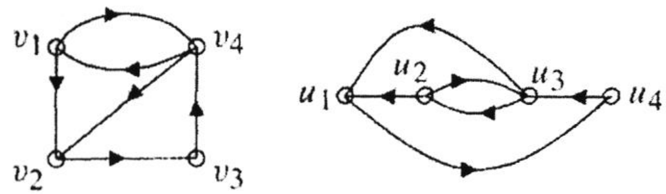


(c) Define adjacency matrix of a graph. Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 given below. Find a permutation matrix P such that $A_2 = PA_1P^T$, thus proving that G_1 and G_2 are isomorphic.



G_1 G_2 (7.5)

3. (a) Write the definition of a digraph along with an example with 5 vertices. Explain whether the following digraphs are isomorphic or not :



(7.5)

(b) Write the definition of a transitive tournament along with an example. Show that if T is a tournament having a unique Hamiltonian path, then T is transitive. (7.5)

(c) The construction of a certain part in an automobile engine involves four activities : pouring the mold, calibration, polishing, and inspection. The mold is poured first; calibration must occur before the inspection. Pouring the mold takes eight units of time, calibration takes three units, polishing takes six units of time for an uncalibrated product and eight units of time for a calibrated one, and inspection takes two units of time for a polished product and three units of time for an unpolished one.

(i) Draw the appropriate directed network that displays the completion of this job.

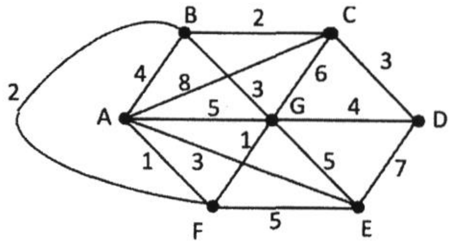
(ii) What is the shortest time required for this job? Describe the critical path.

(3,4.5)

4. (a) (i) Draw all non-isomorphic trees containing 6 vertices.

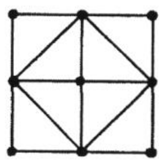
(ii) Solve the Travelling Salesman's problem for the following graph by making tree that displays all the Hamiltonian Cycles (Start with A) :

2. (a) Apply the improved version of Dijkstra's algorithm to find the length of a shortest path from A to D in the graph shown below. Also find the corresponding shortest path. Label all vertices and write steps.



(7.5)

(b) Define Eulerian graph and Hamiltonian graph. Consider the graph G given below. Is it Eulerian? Is it Hamiltonian? Explain your answers.



(7.5)