(c) Use the Runge-Kutta Method fourth-order, determine the approximate solution of the initial value Problem $\frac{dx}{dt} = \frac{x}{t}$, $x(0) = 1$, $1 \le t \le 2$, taking the step size as $h = 0.5$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4140 **H**

Unique Paper Code : 2352012403

Name of the Paper : NUMERICAL ANALYSIS

Name of the Course : **B.Sc. (H) Mathematics – DSC**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All **six** questions are compulsory, attempt any **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of non-programmable Scientific Calculator is allowed.

1. (a) Determine a formula which relates the number of iterations n, required by the bisection method to converge to within an absolute error tolerance ε, starting from the initial interval (a, b).

(b) Perform up to four iterations using the false position to approximate a root of $f(x) = e^x + x^2 - x - 4$ in the interval $(-2, -1)$.

(c) Verify that $x = \sqrt{a}$ is a fixed point of the function $g(x) = \frac{1}{2}\left(x + \frac{a}{x}\right)$. Use the fixed point iteration scheme to determine the order of convergence and the asymptotic error constant of the sequence $p_n = g(p_{n-1})$ towards $x = \sqrt{a}$.

(c) Derive the formula for the Trapezoidal rule and hence evaluate the approximate value of the integral $\int_1^2 \frac{1}{x} dx$.

6. (a) Determine the values for the coefficients A_0, A_1, and A_2 so that the quadrature formula

$$I(f) = \int_{-1}^{1} f(x)\,dx = A_0 f\left(\frac{-1}{3}\right) + A_1 f\left(\frac{1}{3}\right) + A_2 f(1)$$

has degree of precision at least 2.

(b) Use the Euler Method, determine the approximate solution of the initial value problem $\frac{dx}{dt} = 1 + \frac{x}{t}$, $x(1) = 1$, $1 \le t \le 2$, taking the step size as $h = 0.5$.

x	0	1	2	3	4
$f(x)$	1	2	5	10	16

Hence estimate the value of f(1.5), f(2.5) and f(3.5).

5. (a) Prove that $f'(x_0) \approx \dfrac{f(x_0+h)-f(x_0)}{h}$ and determine the approximate value of the derivative of $f(x) = 1 + x + x^3$ at $x_0 = 1$, taking h = 1, 0.1, 0.01 and 0.001.

(b) Verify that the second-order forward difference approximation for the first derivative provides the exact value of the derivative, regardless of h, for the functions f(x) = 1, f(x) = x and $f(x) = x^2$, but not for the function $f(x) = x^3$.

2. (a) Calculate the fourth approximation of the root of the function $f(x) = e^{-x} - x$ using the Newton's method with $p_0 = 0$.

(b) Use the Secant method to calculate the root of the function $f(x) = 1.05 - 1.04x + \ln x$ using $p_0 = 1.10$ and $p_1 = 1.15$ with an absolute tolerance of 10^{-6} as a stopping condition.

(c) Determine the order of convergence of the Newton's method to find a root p of a twice differentiable function f on the interval [a, b], provided $f'(p) \neq 0$.

3. (a) Solve the following system of equations by using the LU Decomposition method

$$x + 4y + 3z = -4,$$

$$2x + 7y + 9z = -10,$$

$$5x + 8y - 2z = 9.$$

(b) Starting with initial vector $(x, y, z, t) = (0, 0, 0, 0)$, perform three iterations of Gauss-Seidel method to solve the following system of equations

$$4x + y + z + t = -5,$$

$$x + 8y + 2z + 3t = 23,$$

$$x + 2y - 5z = 9,$$

$$-x + 2z + 4t = 4.$$

(c) Compute T_{jac} and T_{gs} for the matrix $\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$.

Will the Jacobi and Gauss Seidel method converge for any choice of initial vector $x^{(0)}$? Justify your answer.

4. (a) Find the Lagrange interpolation polynomial for the data set (0,1), (1,3) and (3,55). Also estimate the value at $x = 2.5$.

(b) The population of a town during the last six censuses are given below. Estimate the increase in the population from 1846 to 1848 by using the Newton interpolation polynomial.

Year	1811	1821	1831	1841	1851	1861
Population (in thousands)	9	12	20	17	37	42

(c) Obtain the piecewise linear interpolating polynomial for the function f(x) defined by the given data :