(b) Represent the function $f(z)=\frac{z+1}{z-1}$.

(i) by its Maclaurin series and state the domain where the representation is valid.

(ii) by its Laurent series in the domain $1<|z|<\infty$. (3+3.5)

(c) Find the residue at $z = 0$ of the functions :

(i) $\frac{1}{z+z^2}$ (ii) $z\cos(1/z)$ (3+3.5)

6. (a) State Cauchy Residue Theorem and use it to evaluate the integral of $\frac{1}{1+z^2}$ around the circle $|z| = 2$ in the positive sense. (7)

(b) Show that the point $z = 0$ is a pole of the function $z(e^z - 1)$. Also find order of the pole and corresponding residue. (7)

(c) In each case, write the principal part of the function at its isolated singular point and determine the type of singular point with full explanation :

(i) $e^{1/z}$ (ii) $\frac{z^2}{1+z}$ (3.5+3.5)

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2933 **H**

Unique Paper Code : 32351601

Name of the Paper : Complex Analysis (including Practicals)

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Each question consists of **three** parts. Attempt any **two** parts from each question.

1. (a) (i) Find and sketch, showing corresponding orientations, the image of the hyperbola $x^2 - y^2 = c_1$ ($c_1 < 0$) under the transformation $w = z^2$.

(ii) Suppose $(z)=\begin{cases} z/|z| & \text{if } z\neq 0 \\ 0 & \text{if } z=0 \end{cases}$. Check the continuity of f at $z = 0$. Justify your answer. (4+2=6)

(b) (i) Determine the points where $f(z) = (x^3 + 3xy^2 - x) + i(y^3 + 3x^2y - y)$ is differentiable. Is f analytic at those points? Justify your answer.

(ii) Suppose f(z) is analytic. Can $g(z)=\overline{f(z)}$ be analytic? Justify your answer. (4+2=6)

(c) (i) Suppose $f(z) = e^{-x}e^{-iy}$. Show that $f'(z)$ and $f''(z)$ exist everywhere. Hence prove that $f''(z) = f(z)$.

(ii) Show that $\log(i^{1/2}) = (1/2) \log i$.

(3.5+2.5=6)

2. (a) (i) Find all the roots of the equation $\sin z = 5$.

(ii) Show that $|\exp(z^2)| \le \exp(|z^2|)$. (4+2=6)

(b) (i) Show that $\sin \bar{z}$ is not analytic function anywhere.

(ii) Compute $\lim\limits_{z\to\infty} \dfrac{2z^3-1}{z^2+1}$. (5+1=6)

(c) Evaluate $\int_C \frac{dz}{z}$, where C is a positively oriented circle $z = 2e^{i\theta}$ $(-\pi \le \theta \le \pi)$. (6)

3. (a) Let $f(z) = \pi \exp(\pi\bar{z})$ and C is the boundary of the square with vertices at the points 0, 1, 1 + i and i; orientation of C being in the anticlockwise direction. Parametrize the curve C and evaluate $\int_C f(z)dz$. (6)

(b) Let C denote a contour of length L and suppose that a function f(z) is piecewise continuous on C. If M is a non-negative constant such that $|f(z)| \le M$ for all points z on C at which f(z) is defined, then prove that $\left|\int_C f(z)dz\right| \le ML$. (6)

(c) If a function f is analytic at a given point, then prove that its derivatives of all orders are analytic there too. (6)

4. (a) State Cauchy integral formula and its extension.

Evaluate $\oint_C \frac{z+1}{z^4+2iz^3}dz$, where C is the circle $|z| = 1$. (6)

(b) Show that if f is analytic within and on a simple closed contour C and z_0 is not on C, then

$$\int_C \frac{f'(z)}{z-z_0}dz = \int_C \frac{f(z)}{(z-z_0)^2}dz \quad (6)$$

(c) State and prove Fundamental theorem of Algebra. (6)

5. (a) Find limit of the following sequences as $n \to \infty$:

(i) $z_n = \frac{1}{n^3} + i \quad (n = 1,2,....)$

(ii) $w_n = -2 + i\frac{(-1)^n}{n^2} \quad (n = 1,2,...)$ (3+3.5)