(c) Find the payoff from a bull spread created using call options. Also draw the profit diagram corresponding to this trading strategy.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3071 **H**

Unique Paper Code : 32357614

Name of the Paper : DSE-3 MATHEMATICAL FINANCE

Name of the Course : **B.Sc. (Hons) Mathematics CBCS (LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions are compulsory and carry equal marks.

4. Use of Scientific calculator, Basic calculator and Normal distribution tables all are allowed.

1. (a) Define Convexity of a bond and find the relation between the Convexity, Duration and Bond price. How does convexity measure sensitivity of the portfolio?

(b) Consider the three bonds having payments as shown in the table below. They are traded to procure a 12% yield with continuous compounding.

End of year payments	Bond A	Bond B	Bond C
Year 1	1000	500	0
Year 2	1000	500	0
Year 3	1000+10000	500+10000	0+10000

Determine the price and duration of each bond.

(c) Explain Forward Rates. Derive the following relation :

$$R_F = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

where R_F is the forward interest rate for the period between T_1 and T_2. R_1 and R_2 are the zero rates for maturities T_1 and T_2 respectively. What happens when $R_2 > R_1$?

6. (a) Discuss the delta of a European call option and calculate the delta of an at-the-money 6-month European call option on a non-dividend-paying stock when the risk-free interest rate is 8% per annum and the stock price volatility is 30% per annum.

(b) Companies A and B have been offered the following rates per annum on a ₹10 million loan for 5 years :

	Fixed rate	Floating rate
Company A	12.0%	LIBOR+ 0.1%
Company B	14.5%	LIBOR + 0.9%

Company A requires a floating-rate loan; Company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

5. (a) Given that in a risk-neutral world,

$$\ln S_T \sim \phi\left[\ln S_0 + \left(r - \frac{\sigma^2}{2}\right)T, \sigma^2 T\right],$$

where S_T is the stock price at a future time T, S_0 is the current stock price, r is the risk-free rate, σ is the volatility and $\phi(m, v)$ denotes a normal distribution with mean m and variance v. For the given strike price K, find $P(S_T > K)$, the probability that a European call option be exercised in a risk-neutral world.

(b) Show that the Black—Scholes-Merton formulas for call and put options satisfy the put-call parity.

(c) What is the price of a European call option on a non-dividend-paying stock when the stock price is ₹52, the strike price is ₹50, the risk-free interest rate is 12% per annum, the volatility is 30% per annum, and the time to maturity is three months?

(You can use values: $\ln(26/25) = 0.0392$, $\exp(-0.03) = 0.9704$)

2. (a) (i) An investor sells a European call option with strike price of K and maturity T and buys a put with the same strike price and maturity. Describe the investor's position.

(ii) An investor sells a European call on a share for ₹6, the stock price is ₹45 and the strike price is ₹52. Under what circumstances will the seller of the option make a profit? Under what circumstances will the option be exercised?

(b) Write a short note on European put options. Explain the payoffs in different types of put option positions with the help of diagrams.

(c) Explain Hedging. A United States company expects to pay 1 million Euros in 3 months. Explain how the exchange rate risk can be hedged using

(i) A Forward Contract

(ii) An Option.

3. (a) Name the six factors that affect stock option prices. Explain any three of them.

(b) Derive the put-call parity for European options on a non-dividend-paying stock. Use put-call parity to derive the relationship between the gamma of a European call and the gamma of a European put on a non-dividend-paying stock.

(c) Find lower bound and upper bound for the price of a 1-month European put option on a non-dividend-paying stock when the stock price is ₹30, the strike price is ₹34, and the risk-free interest rate is 6% per annum? Justify your answer with no arbitrage arguments, ($e^{-0.005} = 0.9950$).

4. (a) A stock price is currently ₹40. It is known that at the end of one month it will be either ₹42 or ₹38. The risk-free interest rate is 6% per annum with continuous compounding. Consider a portfolio consisting of one short call and A shares of the stock. What is the value of A which makes the

portfolio riskless? Using no-arbitrage arguments, find the price of a one-month European call option with a strike price of ₹39? (You can use exponential value: $e^{0.005} = 1.005$).

(b) A stock price is currently ₹50. Over each of the next two three-month periods it is expected to go up by 6% or down by 5%. The risk-free interest rate is 5% per annum with continuous compounding. What is the value of a six-month European call option with a strike price of ₹51? (You can use exponential value: $e^{-0.0125} = 0.9876$).

(c) Consider a two-period binomial model with current stock price S_0 = ₹100, the up factor $u = 1.2$, the down factor $d = 0.8$, $T = 1$ year and each period being of length six months. The risk-free interest rate is 5% per annum with continuous compounding. Construct the two-period binomial tree for the stock. Find the price of an American put option with strike K = ₹104 and maturity $T = 1$ year. ($e^{-0.025} = 0.9753$)