(c) The marks obtained by a number of students for a certain subject are assumed to be approximately normally distributed with mean 65 and standard deviation of 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70? (Use $P(-1 < Z < 1) = 0.6826$, $P(-2 < Z < 2) = 0.9544$ and $P(-3 < Z < 3) = 0.9973$) (5,5,5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6094 **H**

Unique Paper Code : 2374001201

Name of the Paper : GE 2 (a) – Introductory Probability

Name of the Course : G.E

Semester : II (NEP)

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section A** is compulsory.
3. Attempt any **five** questions from **Section B**.
4. Use of a non-programmable scientific calculator is allowed.

Section A

1. Answer the following :

(a) Name a discrete distribution and a continuous distribution for which variance < mean.

(b) Let $X \sim \exp(4)$. Find $P(4 \le X \le 6)$.

(c) If X and Y are independent events and $E(X) = 8$, $E(Y) = 10$, then find the value of $E(XY)$.

(d) If $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cup B) = 0.4$ then, find $P(\bar{A} \cap \bar{B})$.

(e) If $M_X(t) = (0.3 + 0.7e^t)^5$, find $E(X)$.

(f) Let X be normally distributed with mean 12 and standard deviation 4, find $P(X \le 30)$.

(g) If $\mu_X = 10$, $\mu_Y = 20$, $\sigma_X^2 = 25$, $\sigma_Y^2 = 36$ and $\text{cov}(X,Y) = 100$, then find $\text{var}(4X-3Y)$.

(h) If $M_X(t) = \left(\frac{1}{4} + \frac{3}{4}e^t\right)^4$, find Var(X).

(i) Define discrete and continuous sample space with examples.

7. (a) A car hire firm has two cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which (i) neither car is used, and (ii) the proportion of days on which some demand is refused. (Use value of $e^{1.5} = 4.48$)

(b) Define exponential distribution and obtain its moment generating function. Hence, find its mean and variance. (7,8)

8. (a) Define Negative Binomial distribution. Give an example in which it occurs? Show that negative binomial distribution may be regarded as the generalization of geometric distribution.

(b) If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try?

(b) If X follows binomial distribution with parameters n and p. Find the value of p, if n=6 and 9P(X=4) = P(X=2). (8,7)

6. (a) Mention five chief characteristics of the normal probability distribution. Also, write its p.d.f.

(b) There are six hundred Economics students in the post graduate classes of a university, and the probability for any student to need a copy of a particular book from the university library on any day is 0.05. How many copies of the book should be kept in the university library so that the probability may be greater than 0.90 that none of the students needing a copy from the library has to come back disappointed? (Use normal approximation to the binomial distribution and $P(0 < Z < z_1) = 0.40$, $z_1 = 1.28$) (8,7)

(j) Let $f(x) = \frac{x^2}{30}$, for x = 0, 1, 2, 3, 4. Determine whether f(x) can serve as the probability distribution of a random variable. (1×5,2×5)

Section B

2. (a) Define independent events with a suitable example. A problem in Statistics is given to three students A, B and C whose chances of solving it are ½, ¾ and ¼ respectively. What is the probability that the problem will be solved, if all of them try independently?

(b) For married couples living in a certain suburb, the probability that the husband will vote in a school board election is 0.21, the probability that the wife will vote in the election is 0.28, the probability that they will both vote is 0.15.

(i) What is the probability that at least one of them will vote?

(ii) What is the probability that none of them will vote? (9,6)

3. (a) State the three properties of the moment generating function. If a random variable X has the probability distribution $f(x)=\frac{1}{8}\binom{3}{x}$ for x = 0, 1, 2 and 3, find the moment generating function of the random variable X and use it to determine μ_1' and μ_2'.

(b) A box contains 20 fuses, of which five are defective. If three of the fuses are selected at random and are removed from the box in succession, what is the probability that all are defective? (9,6)

4. (a) A random variable X has a probability density function given by :

$f(x) = kx^2(1 - x^3)$; $0 \le x \le 1$, where k is a constant

Find the value of k. Hence, obtain its mean and variance.

(b) The amount of bread (in hundreds of pounds) x that a certain bakery is able to sell in a day is found to be a random phenomenon with a probability density function given by :

$$f(x)=\begin{cases} kx, & 0\le x<5 \\ k(10-x), & 5\le x<10 \\ 0, & \text{otherwise} \end{cases}$$

(i) Find the value of k such that f(x) is a probability density function.

(ii) What is the probability that the number of pounds of bread that will be sold tomorrow is more than 500 pounds?

(iii) What is the probability that the number of pounds of bread that will be sold tomorrow is less than 500 pounds? (7,8)

5. (a) Define Uniform distribution. Obtain moments of the uniform distribution, and hence find its mean and variance.