

(b) A paint manufacturer wants to determine the average drying time of new interior wall paint. If for 12 test areas of equal size he obtained a mean drying time of 66.3 minutes and a standard deviation of 8.4 minutes, construct a 95% confidence interval for the population mean μ .

(Given that $z_{0.025} = 1.96$, $t_{0.025,11} = 2.201$.) (9,6)

7. (a) Explain the following terms :

- (i) Statistical hypothesis,
- (ii) MP critical region, and
- (iii) UMP critical region.

(b) In a Bernoulli distribution with parameter p , $H_0: p = \frac{1}{2}$ against $H_1: p = \frac{3}{4}$ is rejected, if more than 4 heads are obtained out of 6 throws of a coin. Find the probabilities of type I and type II errors and the power of the test. (6,9)

8. Write short notes on any **three** :

- (i) Confidence interval for large samples
- (ii) Method of minimum variance
- (iii) Method of least square
- (iv) Neymann-Pearson Lemma (15)

(300)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5912

H

Unique Paper Code : 2374002004

Name of the Paper : Basics of Statistical Inference

Name of the Course : **Statistics: Generic Elective under NEP-UGCF**

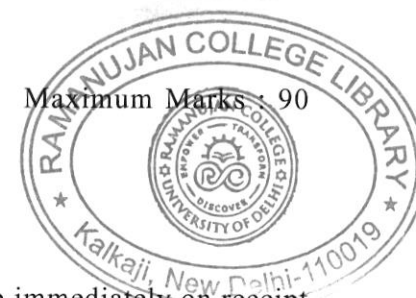
Semester : IV

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Six** questions in all selecting **Three** from each section.
3. Attempt all parts of question in continuation.
4. Use of non-programmable scientific calculator is allowed.



P.T.O.

Section I

1. (a) Define unbiasedness and consistency of the estimators. Give an example of an estimator: (i) which is consistent but not unbiased, and (ii) which is unbiased but not consistent.
- (b) Let X_1, X_2, X_3 be independent variables such that each X_i has mean μ and variance σ^2 . If T_1, T_2 and T_3 are the estimators used to estimate μ , verify whether (i) $T_1 = X_1 + X_2 - X_3$, (ii) $T_2 = (X_1 + X_2 + X_3)/3$, and (iii) $T_3 = (2X_1 - X_2 + X_3)/2$ are unbiased estimators for μ . Which one of these estimators is more efficient? Find efficiency of other estimators. (6,9)
2. State the Cramer-Rao inequality for the variance of an unbiased estimator clearly mentioning the underlying regularity conditions. Further stating the condition for the equality sign in the Cramer-Rao inequality to hold, obtain its form. Verify that there exists an MVB estimator for the parameter θ of the distribution

$$f(x, \theta) = \frac{e^{-\theta} \theta^x}{x!}; x = 0, 1, 2, \dots$$

and hence, obtain the value of MVB estimator.

(15)

3. (a) Explain the sufficient statistic. State the factorization theorem on sufficiency.

- (b) Define completeness of a statistic. Let X_1, X_2, \dots, X_n be a random sample of size n from a Bernoulli distribution

$$f(x, \theta) = \theta^x (1 - \theta)^{1-x}, x = 0, 1.$$

Show that $T = \sum X_i$ is a complete sufficient statistic for θ . (5,10)

4. Write short notes on any **three** :
- (i) Invariance property of a consistent estimator
 - (ii) MVU estimator
 - (iii) Rao-Blackwell Theorem
 - (iv) Lehmann-Scheffe Theorem (15)

Section II

5. (a) Describe the method of maximum likelihood estimation. Obtain the MLE for the parameter θ in a random sample of size n from the Uniform population $U(0, \theta)$.
- (b) State optimum properties of maximum likelihood estimators. (9,6)
6. (a) Critically examine how interval estimation differs from point estimation. Obtain the 95% confidence interval for the variance σ^2 of normal population with mean μ and variance σ^2 .