(b) A paint manufacturer wants to determine the average drying time of new interior wall paint. If for 12 test areas of equal size he obtained a mean drying time of 66.3 minutes and a standard deviation of 8.4 minutes, construct a 95% confidence interval for the population mean μ.

(Given that $z_{0.025} = 1.96$, $t_{0.025,11} = 2.201$.) (9,6)

7. (a) Explain the following terms :

   (i) Statistical hypothesis,

   (ii) MP critical region, and

   (iii) UMP critical region.

   (b) In a Bernoulli distribution with parameter p, $H_0 : p = ½$ against $H_1 : p = ¾$ is rejected, if more than 4 heads are obtained out of 6 throws of a coin. Find the probabilities of type I and type II errors and the power of the test. (6,9)

8. Write short notes on any **three** :

   (i) Confidence interval for large samples

   (ii) Method of minimum variance

   (iii) Method of least square

   (iv) Neymann-Pearson Lemma (15)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5912 **H**

Unique Paper Code : 2374002004

Name of the Paper : Basics of Statistical Inference

Name of the Course : **Statistics: Generic Elective under NEP-UGCF**

Semester : IV

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **Six** questions in all selecting **Three** from each section.

3. Attempt all parts of question in continuation.

4. Use of non-programmable scientific calculator is allowed.

Section I

1. (a) Define unbiasedness and consistency of the estimators. Give an example of an estimator: (i) which is consistent but not unbiased, and (ii) which is unbiased but not consistent.

   (b) Let X_1, X_2, X_3 be independent variables such that each X_i, has mean μ and variance σ^2. If T_1, T_2 and T_3 are the estimators used to estimate μ, verify whether (i) $T_1 = X_1 + X_2 - X_3$, (ii) $T_2 = (X_1 + X_2 + X_3)/3$, and (iii) $T_3 = (2X_1 - X_2 + X_3)/2$ are unbiased estimators for μ. Which one of these estimators is more efficient? Find efficiency of other estimators. (6,9)

2. State the Cramer-Rao inequality for the variance of an unbiased estimator clearly mentioning the underlying regularity conditions. Further stating the condition for the equality sign in the Cramer-Rao inequality to hold, obtain its form. Verify that there exists an MVB estimator for the parameter 0 of the distribution

$$f(x,\theta) = \frac{e^{-\theta}\theta^x}{x!}; x = 0, 1, 2, \ldots.$$

and hence, obtain the value of MVB estimator. (15)

3. (a) Explain the sufficient statistic. State the factorization theorem on sufficiency.

   (b) Define completeness of a statistic. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a Bernoulli distribution

$$f(x,\theta) = \theta^x(1-\theta)^{1-x},\ x = 0, 1.$$

   Show that $T = \Sigma X_i$ is a complete sufficient statistic for θ. (5,10)

4. Write short notes on any **three** :
   (i) Invariance property of a consistent estimator
   (ii) MVU estimator
   (iii) Rao-Blackwell Theorem
   (iv) Lehmann-Scheffe Theorem (15)

Section II

5. (a) Describe the method of maximum likelihood estimation. Obtain the MLE for the parameter θ in a random sample of size n from the Uniform population $U(0, \theta)$.

   (b) State optimum properties of maximum likelihood estimators. (9,6)

6. (a) Critically examine how interval estimation differs from point estimation. Obtain the 95% confidence interval for the variance σ^2 of normal population with mean μ and variance σ^2.