7. (a) If the random variables X_i (i = 1,2,3,4) are independently and identically distributed with probability density function

$$f(x)=\frac{1}{\sqrt{72\pi}}\exp\left[-\frac{(\mu+7)^2}{72}\right], \quad -\infty<x<\infty$$

then obtain the probability density function of $U=\frac{1}{4}\sum_{i=1}^{4}(x_i)$ and $V = X_1 - 2X_2 + 3X_3 - 4X_4$.

(b) If X and Y are independent variates such that $x \sim N(6,9)$ and $Y \sim N(7,16)$ then, determine λ such that $P(2X + Y \leq \lambda) = P(4X - 3Y \leq \lambda)$.

(6,6)

8. (a) Find the probabilities that the value of a random variable will exceed 4 if it has a gamma distribution with (i) $\alpha = 2$ and $\beta = 3$; (ii) $\alpha = 3$ and $\beta = 4$.

(b) Show that the parameters of the beta distribution can be expressed as follows in terms of the mean and the variance of this distribution :

$$\alpha=\mu\left[\frac{\mu(1-\mu)}{\sigma^2}-1\right]$$

$$\beta=(1-\mu)\left[\frac{\mu(1-\mu)}{\sigma^2}-1\right]$$

(6,6)

8/6/24 (Eve)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1066 **H**

Unique Paper Code : 32375201

Name of the Paper : Introductory Probability (GE-II)

Name of the Course : **Generic Elective in Statistics**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section I** is compulsory.
3. Attempt any **five** questions, selecting at least **two** questions from each of the **Sections II** and **III**.
4. Use of simple calculator is allowed.

Section - I

1. (a) Define axiomatic probability.

(b) If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.3$, then find $P(A|B)$ and $P(B|A)$.

(c) Consider a random variable X having probability mass function $f(x)=\frac{x}{10}$ for x = 1,2,3,4. Find the mean of X.

(d) If $X_n \xrightarrow{p} \alpha$ and $Y_n \xrightarrow{p} \beta$ as $n \to \infty$ then

(i) $X_n \pm Y_n \xrightarrow{p}$ ______ as $n \to \infty$ and

(ii) $X_n Y_n \xrightarrow{p}$ ______ as $n \to \infty$.

(e) If $M_x(t)$ is the m.g.f of a random variable X, then $M_{x+a}(t) =$ ______ and $M_{bx}(t) =$ ______, where a and b are constants.

(f) For each of the following, determine whether the given function can serve as the probability distribution of a random variable with the given range :

(i) If he fires 7 times, what is the probability of his hitting the target at least twice?

(ii) How many times must he fire so that the probability of his hitting the target at least once is greater than $\frac{2}{3}$.

(c) Suppose that X has Poisson distribution. If $P(X=2) = \frac{2}{3}P(X=1)$, evaluate $P(X=0)$ and $P(X=3)$. (4,4,4)

6. (a) Suppose X has the geometric distribution with p.m.f. $f(x) = q^{x-1} p$, x = 1,2,3,... then, show that the moment generating function is $M_x(t) = \frac{pe^t}{1-qe^t}$.

Also, find $M'_X(0)$ and $M''_X(0)$. Hence, compute the mean and variance of random variable X.

(b) Let X have the pdf $f(x) = e^{-x-1}$, $-1 < x < \infty$. Find $(X \geq 1)$, and show that the moment generating function is given by $M_x(t) = e^t(1-t)^{-1}$. Also, find $M'_x(0)$ and $M''_x(0)$. Hence, compute the mean and variance of the random variable X. (6,6) defective. What is the probability that it comes from the output (i) Machine I, (ii) Machine II, and (iii) Machine III?

(b) Let X be a continuous random variable with pdf

$$f(x)=\begin{cases} ax, & 0\le x\le 1 \\ a, & 1\le x\le 2 \\ -ax+3a, & 2\le x\le 3 \\ 0, & \text{elsewhere} \end{cases}$$

(i) Determine the constant a,

(ii) Determine F(x), the cdf, and sketch its graph. (6,6)

Section – III

5. (a) The p.d.f. of the random variable X is given by

$$f(x)==\begin{cases} \frac{c}{\sqrt{x}}, & \text{for } 0<x<4 \\ 0, & \text{elsewhere} \end{cases}$$

Find,

(i) the value of c.

(ii) $P\left(X<\frac{1}{4}\right)$ and P (X > 1).

(b) The probability of a man hitting a target is $\frac{1}{4}$.

(i) $f(x)=\frac{x-2}{5}$ for x = 1,2,3,4,5.

(ii) $f(x)=\frac{x^2}{30}$ for x = 0,1,2,3,4.

(g) If X and Y are independent r.v.s with $\mu_x = 3$, $\mu_y = 5$, $\sigma_x^2 = 8$ and $\sigma_Y^2 = 12$, then obtain mean and variance of U = X + 3Y.

(h) If X has the probability density $f(x) = e^x$ for x > 0 and 0, elsewhere, then find the expected value of $g(x) = e^{3X/4}$.

(i) Let μ and σ^2 denote the mean and variance of the random variable X. Determine $E\left(\frac{X-\mu}{\sigma}\right)$ and $E\left(\frac{X-\mu}{\sigma}\right)^2$.

(j) Subway trains on a certain line run every half hour between midnight and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least 20 minutes? (1×5,2×5)

Section – II

2. (a) A random variable X has the following probability mass function.

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

(i) Find k.

(ii) Evaluate $P(X < 6)$.

(iii) If $P(X \leq a) > \frac{1}{2}$, then find the minimum value of a.

(b) A continuous random variable X has the p.d.f. $f(x) = 3x^2$, $0 \leq x \leq 1$. Find a and b such that (i) $P(X \leq a) = P(X > a)$, and (ii) $P(X > b) = 0.05$.

(c) If X is the number scored in a throw of a fair die, show that the Chebychev's inequality gives $P\{|x - \mu| > 2.5\} < 0.47$, where μ is the mean of X, while the actual probability is zero.

(4,4,4)

3. (a) A coin is tossed three times, and the outcomes are assumed to be equally likely. If A is the event that a head occurs on each of the first two tosses, B is the event that a tail occurs on the third toss, and C is the event that exactly two tails occur in the three tosses, show that (i) events A and B are independent; (ii) events B and C are dependent.

(b) Show that if a random variable has an exponential density with the parameter θ, the probability that it will take a value less than $-\theta \ln(1 - p)$ is equal to p for $0 \leq p < 1$.

(c) If the probability density of X is given by

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x \leq 1 \\ \frac{1}{2}, & 1 < x \leq 2 \\ \frac{3-x}{2}, & 2 \leq x < 3 \\ 0, & \text{elsewhere} \end{cases}$$

Find the expected value of $g(X) = X^2 - 5X + 3$.

(4,4,4)

4. (a) A factory produces a certain type of output by three types of machines. The respective daily produce figures are :

Machine I: 3,000 Units, Machines II: 2,500, Machine III: 4,500 Units.

Past experience shows that 1% of the output produced by machine I is defective. The fractions of defective for the other two Machines are 1.2% and 2% respectively. An item is drawn at random from the day's production run and is found to be