(c) Let W be the subspace of $\mathbb{R}^3$, whose vectors lie in the plane $2x - 2y + z = 0$ and $v = [1, -4, 3] \in \mathbb{R}^3$. Find $\text{proj}_W v$, and decompose v into $w_1 + w_2$, where $w_1 \in W$ and $w_2 \in W^{\perp}$. (6)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 1047 **H**

Unique Paper Code : 32355202

Name of the Paper : Linear Algebra

Name of the Course : **Generic Elective-Mathematics [other than Maths (H)]**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** questions by selecting any **two** parts from each question.

P.T.O.

1. (a) If x and y are non-zero vectors in R^n, then prove that (6½)

(i) If x, y $\geq$ 0, then $||x + y|| > ||y||$.

(ii) $||x + y||^2 + ||x - y||^2 = 2(||x||^2 + ||y||^2)$.

(b) If x and y are non-zero vectors in $\mathbb{R}^n$, then x, y = $||x||\ ||y||$ if and only if y is a positive scalar multiple of x. (6½)

(c) Reduce the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 6 \\ 3 & 8 & 9 & 10 \\ 2 & -1 & 2 & -2 \end{bmatrix}$ to reduced row echelon form by using elementary row operations. (6½)

(b) Find the kernel and range of the linear transformation $L : \mathbb{R}^4 \to \mathbb{R}^4$ defined by $L([a_1, a_2, a_3, a_4]) = [0, a_2, 0, a_4]$. (6½)

(c) Check whether the linear transformation $L: \mathbb{R}^2 \to \mathbb{R}^2$ given by

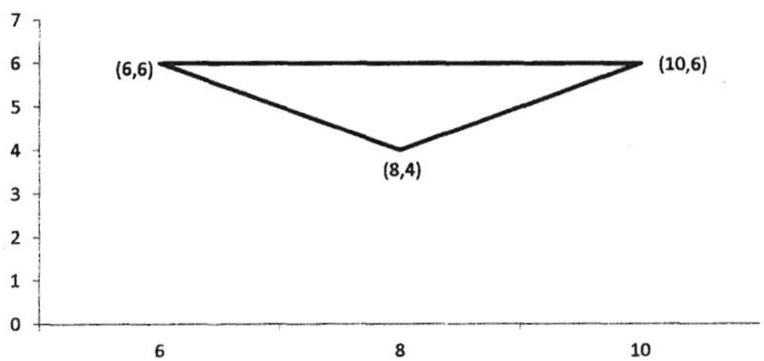
$$L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -4 & -3 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

is one to one or onto. (6½)

6. (a) Find a least square solution for the following inconsistent system (6)

$$\begin{pmatrix} 2 & 3 \\ 1 & -1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 4 \end{pmatrix}$$

(b) Let W = span{[1, –2, –1], [3, –1, 0]}. Find a basis for W and its orthogonal complement $W^{\perp}$. (6)



(i) Translation along the vector (–3,5).

(ii) Scaling about the origin with scale factors of $\frac{1}{2}$ in the x-direction and 3 in the y-direction. (6½)

2. (a) Solve the following system of linear equations using Gauss-Jordan method

$$2x_1 + 3x_2 - x_3 = 9$$

$$x_1 + x_2 + x_3 = 9$$

$$3x_1 - x_2 - x_3 = -1$$ (6)

(b) Show that the matrix $A = \begin{bmatrix} -3 & -1 & -2 \\ -2 & 16 & -18 \\ 2 & 9 & -7 \end{bmatrix}$ is not diagonalizable. (6)

(c) Prove that the set S = {[1, 3, –1], [2, 7, –3], [4, 8, –7]) spans $\mathbb{R}^3$. (6)

3. (a) Use the Simplified Span Method to find a simplified general form for all the vectors in span(S) where S = {[1, –1, 1], [2, –3, 3], [0, 1, –1]} is a subset of $\mathbb{R}^3$. Is the set S linearly independent? Justify. (6½)

(b) Examine whether the set $S = \{[1, 3, -1], [2, 7, -3], [4, 8, -7]\}$ forms a basis for $\mathbb{R}^3$? (6½)

(c) Define rank of a matrix. Let $A = \begin{bmatrix} -2 & 1 & 8 \\ 7 & -2 & -22 \\ 3 & -1 & -10 \end{bmatrix}$.

Using rank of A determine whether the homogeneous system $AX = 0$ has a non-trivial solution or not. If so, find the non-trivial solution. (6½)

4. (a) Let $S = \{[1, -1], [2, 1]\}$ and $T = \{[3,0], [4, -1]\}$ be two ordered bases for $\mathbb{R}^2$. Find the transition matrix $P_{S \leftarrow T}$ from T-basis to S-basis. Hence, or otherwise find transition matrix from S-basis to T-basis. (6)

(b) Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear operator and $L([1, 0, 0]) = [1, 1, 0]$, $L([0, 1, 0]) = [2, 0, 1]$ and $L([0, 0, 1]) = [1, 0, 1]$. Find $L([x,y,z])$ for any $[x,y,z] \in \mathbb{R}^3$. Also find $L([1, 2, 3])$. (6)

(c) Let $L: \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by $L([x, y, z)] = [-2x + 3z, x + 2y - z]$. Let $B = \{[1, -3, 2], [-4, 13, -3], [2, -3, 20]\}$ and $C = \{[-2, -1], [5, 3]\}$ be a basis for $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively. Find the matrix A_{BC} for L with respect to the given basis B and C. (6)

5. (a) For the graphic in the following figure, use ordinary coordinates in $\mathbb{R}^2$ to find new vertices after performing each indicated operation. Then sketch the figures that would result from each movement.