$L([x,y,z]) = [-2x + 3z,\ x + 2y - z]$.

Find the matrix representation for L with respect to the basis

B = {[1,–3,2], [–4,13,–3], [2,–3,20]} and C = {[–2,–1], [5,3]} for $\mathbb{R}^3$ and $\mathbb{R}^2$ respectively.



22/05/2024 (Evening)

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 6081 H

Unique Paper Code : 2354001202

Name of the Paper : Introduction to Linear Algebra

Name of the Course : GE II

Students : Regular

Semester : II

Duration : 3 Hours Maximum Marks : 90



**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **all** question by selecting two parts from each question.

3. **All** questions carry equal marks.

1. (a) Find the angle between the vectors x = [8, –20, 4], y = [6, –15, 3]. Are vectors x and y parallel? Justify your answer. Also, verify the answer by finding $c \in \mathbb{R}$ such that x = cy.

(b) Use the Gaussian elimination method to find the values of the scalars a, b and c which satisfy the quadratic equation $y = ax^2 + bx + c$ that passes through the points (3,20), (2,11) and (–2,15) in the xy-plane.

(c) Use the Gauss-Jordan method to convert the augmented matrix to reduced row echelon form:

$$\left[\begin{array}{ccc|c} 3 & 1 & -2 & 1 \\ 4 & 0 & -1 & 7 \\ 2 & -3 & 5 & 18 \end{array}\right].$$

2. (a) Let x and y be vectors in $\mathbb{R}^n$ then prove that $||x + y||^2 + ||x - y||^2 = ||x||^2 + ||y||^2$ if and only if x. y = 0.



$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 3 & 2 & 11 \\ 2 & 1 & 8 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(i) Is [–5, 2, 1] in ker(L)? Justify the answer.

(ii) Is [3, 2] in range(L)? Justify the answer.

(b) Let L: $\mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation given by :

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -7 & 4 & -2 \\ 16 & -7 & 2 \\ 4 & -3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Find the dimensions of the kernel of L and the range of L. Use them to determine that whether the linear transformation L is one-one or not, and onto or not.

(c) Let L : $\mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation given by :

(ii) $L_2: P_2 \rightarrow P_3$ given by $L_2(p(x)) = x^3p'(0) + x^2p(0) + 1$, where $p'(x)$ denotes the derivative of $p(x)$ with respect to $x$.

(b) Let $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a linear transformation such that :

$L([1,0,0]) = [0,-2,1]$, $L([0,1,0]) = [-1,1,2]$, $L([0,0,1]) = [2,3,-3]$.

Find $L([2, -3, 1])$. Give a formula for $L([x, y, z])$ for any $[x, y, z] \in \mathbb{R}^3$.

(c) Let $L: M_{3\times3} \rightarrow \mathbb{R}$ be a linear transformation given by $L(A) = \text{trace}(A)$. Find a basis for ker(L) and range(L). Also, verify that $\dim(\ker(L)) + \dim(\text{range}(L)) = \dim(M_{3\times3})$.

6. (a) Let $L: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation given by :



(b) Express the vector $x \in \mathbb{R}^3$ as a linear combination of the other vectors (if possible)

$x = [5,9,5]$, $a_1 = [2,1,4]$, $a_2 = [1,-1,3]$, $a_3 = [3,2,5]$.

(c) Show that the vector $x = [5, 17, 20]$ is in the row space of the given matrix

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}.$$

3. (a) Find the characteristic polynomial and eigen values of the matrix :

$$A = \begin{bmatrix} 4 & 0 & -2 \\ 6 & 2 & -6 \\ -2 & 0 & 1 \end{bmatrix}.$$

Find the eigen spaces corresponding to the eigen values of A.

(b) Let $V = \mathbb{R}^2$ be a vector space with respect to the operations of addition and scalar multiplication, defined as, for $[x,y]$, $[z,w] \in V$ and $a \in \mathbb{R}$ :

$[x,y] \oplus [w,z] = [x + w - 2,\ y + z + 3]$ and $a \odot [x,y]$

$= [ax - 2a + 2,\ ay - 3a - 3]$.

Prove the closure and associative properties with respect to the addition operation in V. Also, find the zero vector and the additive inverse of each vector in V.

(c) Consider the subset $S = \{[3, 1, -1], [5, 2, -2], [2, 2, -1]\} \subset \mathbb{R}^3$. Use the simplified Span method to find the span(S). Also, find the dimension of span(S).

4. (a) Use the Independence test method to determine whether the given set

$S = \{[2, 5, -1, 6], [4, 3, 1, 4], [1, -1, 1, -1]\} \subset \mathbb{R}^4$

is linearly independent or linearly dependent in $\mathbb{R}^4$.



(b) (i) Check whether the set

$$S = \left\{ \alpha \begin{bmatrix} a & -a \\ b & 0 \end{bmatrix} \in M_{2\times 2} \,\middle|\, \alpha, a, b \in \mathbb{R} \right\}$$

is a subspace of $M_{2\times 2}$ or not, under usual matrix operations.

(ii) Check whether the set $S = \{[1, a] \in \mathbb{R}^2 \mid a \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^2$ or not, under usual operations.

(c) Prove that the set

$$S = \left\{ \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 2 & -2 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 0 & -2 \end{bmatrix} \right\}$$

is a basis for $M_{2\times 2}$.

5. (a) Determine whether the following functions are linear transformations or not :

(i) $L_1 : \mathbb{R}^3 \to \mathbb{R}^3$ given by $L_1([x_1, x_2, x_3]) = [5x_1 - x_3, x_2, 3x_3 - x_2]$.