(ii) $g(x) = x^{1/3}(4 - x)$

(iii) $h(x) = x^2(x - 1)^{2/3}$.

(c) Trace the curve $r = 2 + 4\cos\theta$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2414 **G**

Unique Paper Code : 2354001001

Name of the Paper : GE: Fundamentals of Calculus

Name of the Course : **Common Prog. Group**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **All** questions are compulsory and carry equal marks.

3. This question paper has **six** questions.

4. Attempt any **two** parts from each question.

1. (a) (i) Establish that $\lim_{x\to 0}\frac{e^{1/x}-1}{e^{1/x}+1}$ does not exist.

(ii) Examine the continuity of the function

$$f(x)=\begin{cases} 0 & , \text{ if } x=0,\frac{1}{2} \\ \frac{1}{2}-x & , \text{ if } 0<x<\frac{1}{2} \\ 4x^2-1 & , \text{ if } \frac{1}{2}<x<\frac{3}{4} \\ 1-x^2 & , \text{ if } \frac{3}{4}\le x\le 1 \end{cases}$$

at $x=0,\frac{1}{2},\frac{3}{4}$ and 1. Classify their type of discontinuities, if any.

(b) Find the derivatives of

(i) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$.

(b) Find asymptotes of the curve

$$x^3 - 2y^3 + xy(2x-y) + y(x-y) + 1 = 0.$$

(c) Determine the intervals of concavity and points of inflections of the curve $y=\frac{x^3-x}{3x^2+1}$. Also, show that curve $x^2(x^2+y^2) = a^2(x^2-y^2)$ has no asymptotes.

6. (a) Sketch the graph of the function $y=x^2-\frac{2}{x}$ and also identify the locations of all asymptotes, intercepts, relative extrema and inflection points.

(b) Locate the critical points and identify which critical points are stationary points for the functions

(i) $f(x) = x^3 - 3x + 2$

whether it is applicable to compute the following limits or not.

(i) $\lim_{x \to 1} \left(\frac{2x - 2}{x^3 + x - 2} \right)$

(ii) $\lim_{x \to 0} \left(\frac{\cos x}{x} \right)$

(iii) $\lim_{x \to 0} \left(\frac{e^{2x} - 1}{\tan x} \right)$

5. (a) Determine the intervals of concavity and points of inflections of the curve $y = x^4 - 4x^3 - 18x^2 + 1$. Show that the points of inflection of the curve $y = (x - 2)\sqrt{(x-3)}$ lies on the line $3x = 10$.

(ii) $x^x + (\sin x)^{\log x}$.

(c) Find the n^{th} derivatives of $f(x) = \frac{1}{1 - 5x + 6x^2}$ and $g(x) = \sin 4x \cos 2x$.

2. (a) If $y = \tan^{-1} x$, then prove that

$$(1 + x^2)y_{n+2} + (2n + 2)xy_{n+1} + n(n + 1)y_n = 0.$$

Also find $y_n(0)$.

(b) Let $u = \cot^{-1}\left(\frac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ and $v = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$.

Prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{4}\sin 2u = 0 \text{ and } x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y}\sin 2v.$$

(c) Prove that if $u = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$, then

$$\frac{\partial^2 u}{\partial^2 x} + \frac{\partial^2 u}{\partial^2 y} + \frac{\partial^2 u}{\partial^2 z} = 0.$$

3. (a) Examine the applicability of Rolle's theorem on the following functions :

(i) $f(x) = |x|$ in $[-1,1]$.

(ii) $f(x) = \tan x$ in $[0,\pi]$.

(iii) $f(x) = 10x - x^2$ in $[0,10]$.

(b) State whether the following statements are true or false. Justify your answer.

(i) Lagrange's mean value theorem is not applicable on $f(x) = \frac{1}{x-3}$ in $[5,10]$.

(ii) $\frac{x}{1+x^2} < \tan^{-1} x < x$, for all $x > 0$.

(c) State and prove Cauchy's mean value theorem. Use it to deduce Lagrange's mean value theorem.

4. (a) State Taylor's theorem. Discuss the Lagrange's and Cauchy's form of the remainder of a Taylor's series. Moreover if $f(x) = (1 - x)^{5/2}$, then find the value of c as $x \to 1$ such that

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(cx).$$

(b) Find the Taylor's series for $f(x) = \log(1 + x)$ where $x \in (-1,1)$ and $g(x) = e^x$.

(c) State L'Hopitals rule for form 0/0 for the limits $x \to a$, where a is a real number. Justify that