(c) For given initial value problem (IVP)

$$\frac{dy}{dx} = y - x,\ y(0) = 2,$$

find y(0.5) and y(1.0) by using the Heun's method. Also, find the absolute error at each step, given that the exact solution of the IVP is $y(x) = e^x + x + 1$. (6.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1162 **H**

Unique Paper Code : 32355402

Name of the Paper : GE-4: Numerical Methods

Name of the Course : **CBCS / LOCF (Other than B.Sc. (H) Mathematics Hons.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. Use of scientific calculator is allowed.

1. (a) Convert $(0.7)_{10}$, given in the decimal number system, to the binary number system with 7 significant digits. (6)

(b) Write the approximate value of the number $\frac{5}{3}$ with four significant digits and evaluate round-off error, relative error and absolute percentage error. (6)

(c) Find the approximate root of equation $x^3 + x - 1 = 0$ correct up to three decimal places by using the secant method on the interval (0,1). (6)

2. (a) Find the approximate root of the equation $x^4 - 3x + 1 = 0$ by using Newton-Raphson's method by taking initial approximation $x_0 = 1.5$. Perform three iterations of the method. (6.5)

(b) Find the approximate root of the equation $x^2 - 2x - 1 = 0$ in the interval (2,3) by using the Regula-Falsi method. Perform three iterations of the method. (6.5)

(c) Use the formula

$$f'(x) \approx \frac{-3f(x_i) + 4f(x_i + h) - f(x_i + 2h)}{2h}$$

to approximate the derivative of $f(x) = \ln x$ at $x_i = 2$, taking h = 1, 0.1 and 0.01. (6)

6. (a) Approximate the value of $(\ln 2)^{\frac{1}{3}}$ from $\int_0^1 \frac{x^2}{1+x^3} dx$ by using the Trapezoidal rule with $h = 0.25$. (6.5)

(b) Apply Euler's method to approximate the solution of the initial value problem :

$$\frac{dy}{dx} = \frac{1+y^2}{x}, \quad 1 \le x \le 4, \quad y(1) = 0,$$

by using 5 steps. (6.5)

$$f'(x_i) \approx \frac{f(x_i) - f(x_i - h)}{h}$$ and

$$f''(x_i) \approx \frac{f(x_i - 2h) - 2f(x_i - h) + f(x_i)}{h^2}.$$

x	0.60	0.65	0.70	0.75
$f(x)$	0.6221	0.6155	0.6138	0.6170

(b) Given the function $f(x) = 1 + x + x^3$, approximate $f'(1)$ with Richardson extrapolation by using first order forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$ with $h = 0.25$ and 0.125.

(6)

(c) Find the approximate root of the equation $\cos x - xe^x = 0$ in the interval (0,1) by using the bisection method. Perform four iterations of the method. (6.5)

3. (a) Using Gauss Jordan's method solve the following system of linear equations : (6)

$$x + 2y + z = 8$$

$$2x + 3y + 4z = 20$$

$$4x + 3y + 2z = 16.$$

(b) Show that $1 - E^{-1} = -\frac{1}{2}\delta^2 + \delta\mu$. (Note: Symbols have their own meaning) (6)

(c) Find the Newton interpolating polynomial which fits into the given data. (6)

x	1	2	7	8
f(x)	1	5	5	4

4. (a) By using the initial solution (0,0,0), perform three iterations of the Gauss Jacobi's method for the following system of linear equations : (6.5)

$$27x + 6y - z = 85$$

$$6x + 15y + 2z = 72$$

$$x + y + 54z = 110.$$

(b) Obtain the piecewise linear interpolating polynomial for the function f(x) defined by the given data and by using it estimate the value of f(1.5). (6.5)

x	0	1	2	3
f(x)	1	2	5	10

(c) Following table gives the area of the circle with given diameter : (6.5)

Diameter (in cm)	80	85	90	95	100
Area (in cm square)	5026	5674	6362	7088	7854

Make the difference table. Obtain the backward Gregory-Newton interpolating polynomial and estimate the area of the circle with diameter 82 cm.

5. (a) For the given below data, find f′(0.7) and f″(0.7) by using forward difference formulae (6)