5. (a) Find the solution of the linear partial differential equation

$(y - u)u_x + (u - x)u_y = x - y$, with Cauchy data $u = 0$ on $xy = 1$. (6)

(b) Find the solution of the following partial differential equation by the method of separation of variables

$u_x + u = u_y, \quad u(x, 0) = 4\exp(-3y).$ (6)

(c) Reduce the equation to canonical form and obtain the general solution

$$u_x + 2xy\, u_y = x. \quad (6)$$

6. (a) Find the general solution of the linear partial differential equation

$yz\, u_x - xz\, u_y + xy(x^2 + y^2)\, u_z = 0.$ (6.5)

(b) Reduce the equation

$u_{xx} - \frac{1}{c^2}u_{yy} = 0,\ c \neq 0$ where c is a constant,

into canonical form and hence find the general solution. (6.5)

(c) Reduce the following partial differential equation with constant coefficients,

$$3\, u_{xx} + 10\, u_{xy} + 3\, u_{yy} = 0$$

into canonical form and hence find the general solution. (6.5)

30/12/22(EVE)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3924 **G**

Unique Paper Code : 32355301

Name of the Paper : GE – III Differential Equations

Name of the Course : **Generic Elective / Other than B.Sc. (H) Mathematics**

Semester : III

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.

1. (a) Solve the following differential equations :

$(4x + 3y + 1)dx + (x + y + 1)dy = 0,\ y(3) = -4.$ (6.5)

(b) Solve the Initial Value problem :

$$x^2\frac{dy}{dx} + xy = \frac{y^3}{x}, \quad y(1) = 1 \quad (6.5)$$

(c) Find the orthogonal trajectories of the family of ellipse having centre at the origin , a focus at the point (c,0) and semimajor axis of length 2c. (6.5)

2. (a) (i) Find the Wronskian of the set $\{1 - x, 1 + x, 1 - 3x\}$ and hence find their linear independence or dependence on $(-\infty,\infty)$. (6)

(ii) Solve the differential equation :

$$\frac{dy}{dx} = \frac{2 + ye^{xy}}{2y - xe^{xy}}.$$

(b) Given that $y = x$ is a solution of the differential equation

$$\left(x^2 - x + 1\right)\frac{d^2y}{dx^2} - \left(x^2 + x\right)\frac{dy}{dx} + (x + 1)y = 0,$$

find a linearly independent solution by reducing the order. Write the general solution also. (6)

(c) Solve the following differential equations :

(i) $x\frac{dy}{dx} - 2y = 2x^4,\ y(2) = 8$

(ii) $(2xy^2 + y)dx + (2y^3 - x)dy = 0.$ (6)

3. (a) Solve the Initial value Problem

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 10\frac{dy}{dx} = 0, \qquad y(0) = 7, \qquad \frac{dy}{dx}(0) = 0,$$
$$\frac{d^2y}{dx^2}(0) = 70.$$ (6.5)

(b) Find the general solution of the differential equation using method of Undetermined Coefficients

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 8\sin(3x).$$ (6.5)

(c) Find the general solution of the differential equation using method of Variation of Parameters

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec}(x).$$ (6.5)

4. (a) Given that sin x is a solution of the differential equation

$$\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 0$$

find the general solution. (6)

(b) Find the general solution of the differential equation by assuming $x > 0$,

$$x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 0$$ (6)

(c) Find the general solution of the given linear system

$$2\frac{dx}{dt} + \frac{dy}{dt} - x - y = e^{-t}, \quad \frac{dx}{dt} + \frac{dy}{dt} + 2x + y = e^{t}.$$ (6)