

5. (a) Apply  $\sqrt{u} = v$  and  $v(x, y) = f(x) + g(y)$  to solve

$$x^4 u_x^2 + y^2 u_y^2 = 4u. \quad (7.5)$$

- (b) Find the solution of the initial-value systems

$$u_t + u u_x = e^{-x}v, \quad v_t - av_x = 0,$$

with  $u(x, 0) = x$  and  $v(x, 0) = e^x. \quad (7.5)$

- (c) Determine the general solution of

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2. \quad (7.5)$$

6. (a) Given that the parabolic equation

$$u_{xx} = au_t + bu_x + cu + f,$$

where the coefficients are constants, by the substitution  $u = v e^{\frac{1}{2}bx}$  and for the case  $c = -(b^2/4)$ , show that the given equation is reduced to the heat equation

$$v_{xx} = a v_t + g,$$

where  $g = f e^{-bx/2}. \quad (7.5)$

- (b) Find the solution of the Cauchy problem

$$x u_x - y u_y + y^2 u = y^2. \quad (7.5)$$

- (c) Apply  $v = \ln u$  and then  $v(x, y) = f(x) + g(y)$  to solve

$$x^2 u_x^2 + y^2 u_y^2 = (x y u)^2. \quad (7.5)$$

(1000)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2253

G

Unique Paper Code : 2354002001

Name of the Paper : DIFFERENTIAL EQUATIONS

Name of the Course : COMMON PROG GROUPS  
(Generic Elective)

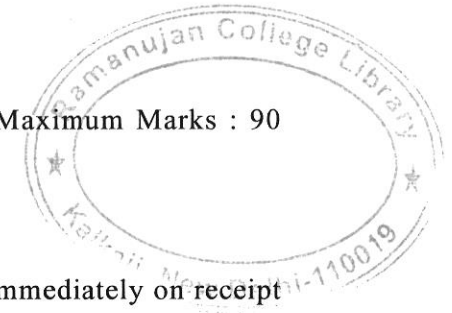
Semester : III

Duration : 3 Hours

Maximum Marks : 90

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.
  2. Attempt **all** question by selecting **two** parts from each question.
  3. **All** questions carry equal marks.
  4. Use of Calculator not allowed.
1. (a) Determine the values of  $p$  for which the function  $g$  defined by  $g(x) = x^p$  is a solution of the differential equation



P.T.O.

$$x^3 \frac{d^3y}{dx^3} + 2x \frac{d^2y}{dx^2} - 10x \frac{dy}{dx} - 8y = 0. \quad (7.5)$$

(b) Solve the equation

$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0. \quad (7.5)$$

(c) Solve the differential equation

$$2r(s^2 + 1)dr + (r^4 + 1)ds = 0. \quad (7.5)$$

2. (a) Solve the differential equation

$$\frac{dx}{dt} + \frac{x}{t^2} = \frac{1}{t^2}. \quad (7.5)$$

(b) Solve the differential equation

$$(5xy + 4y^2 + 1)dx + (x^2 + 2xy)dy = 0,$$

by first finding an integrating factor. (7.5)

(c) Determine the value of K such that the parabolas  $y = c_1x^2 + K$  are the orthogonal trajectories of the family of ellipses  $x^2 + 2y^2 - y = c_2$ . (7.5)

3. (a) Find the solution of the differential equation

$$4 \frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 37y = 0, \quad y(0) = 2, \quad y'(0) = -4. \quad (7.5)$$

(b) Find the general solution of the differential equation

$$y'' - 5y' + 6y = 4e^{2x},$$

using method of undetermined coefficients.

(7.5)

(c) Use the method of variation of parameters to find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = \sec^2 x. \quad (7.5)$$

4. (a) Find the general solution of the differential equation

$$x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 2y = x^3. \quad (7.5)$$

(b) Solve the linear system

$$\frac{dx}{dt} + \frac{dy}{dt} + 2y = \sin t, \quad \frac{dx}{dt} + \frac{dy}{dt} - x - y = 0 \quad (7.5)$$

(c) Show that  $x = 2e^{2t}$ ,  $y = -3e^{2t}$  and  $x = e^{7t}$ ,  $y = e^{7t}$  are two linearly independent solutions on every interval  $a \leq t \leq b$  of the homogeneous linear system

$$\frac{dx}{dt} = 5x + 2y, \quad \frac{dy}{dt} = 3x + 4y$$

Write the general solution. (7.5)