(ii) What will be the probability that dividend is paid in March 2023, given dividend is suspended in January 2023?

(4+5)

[This question paper contains 12 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4152 **H**

Unique Paper Code : 2342011203

Name of the Paper : Probability for Computing

Name of the Course : **B.Sc. (H) Computer Science-DSC**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. The paper has **two** sections. **Section A** is compulsory.
3. Attempt any **four** questions from **Section B**. Each question is of **15** marks.
4. Part of the questions to be attempted together.
5. Use of non-programmable Scientific Calculator allowed.

Section A

1. (a) Let E, F, G be three events. Find expression for the events that of E, F, G

   (i) Both E and F but not G occur

   (ii) At least one events occur

   (iii) At most two occur

   (iv) All three events occur (4)

   (b) (i) When are two states said to communicate with each other?

   (ii) For the given transition probability matrix of a four-state Markov chain with states 0, 1, 2, and 3, answer the following :

$$P=\begin{Vmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 1 \end{Vmatrix}$$

7. (a) Explain the n-step transition probabilities of a Markov chain using Chapman Kolmogorov equations. (6)

   (b) A company pays dividends on a monthly basis when it is earning profits, and suspends the dividend payments in unprofitable times. Suppose that after a dividend has been paid in the current month, the dividend is paid in the next month with probability 0.9, while after a dividend is suspended the next one will be suspended with probability 0.6.

   (i) What is the one-step transitional probability matrix for the above problem?

6. (a) Name and prove the following statement: if X is a random variable that takes only nonnegative values, then show that for any value

$a > 0,\ P\{X \geq a\} \leq \frac{E[X]}{a}$. (4)

(b) Suppose the joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 6xy(2-x-y), & 0<x<1, 0<y<1 \\ 0, & otherwise \end{cases}$$

Compute the conditional expectation of X given that $Y = y$, where, $0 < y < 1$. (6)

(c) Suppose that 5 percent of men and 0.25 percent of women are colour-blind. A colour-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. (5)

(a) Which state is an absorbing state?

(b) Do states 0 and 2 communicate?

(c) Do states 0 and 1 communicate? (2+3)

(c) What is Random number? Write the name of one approach to define the Random number. (2)

(d) There are n components. On a rainy day, component i will function with probability p_i; on a non-rainy day, component i will function with probability q_i, for $i = 1, 2, ..., n$. It will rain tomorrow with probability α. Calculate conditional expected number of components that function tomorrow, given that it rains. (3)

(e) Write down the probability mass function of the following distributions:

(i) Binomial Random variable

(ii) Geometric Random Variable (2+2)

(f) If X and Y are independent, then show that for any function h and g: $E[g(X)h(Y)] = E[g(X)]\, E[h(Y)]$ (3)

(g) Prove the following :

$$Var(X) = E[Var(X|Y)] + Var(E[(X|Y])$$ (4)

(h) Coming home from work, Neha always encounters traffic signal. The probability that she makes it through a traffic signal is 0.2. How many traffic signals can she expect to hit before making it through one? What is the probability of the third traffic light being the first one that is green? (5)

Section B

2. (a) What is Normal Random Variable? Drive the formula of the following :

5. (a) Suppose we know that the number of items produced in a factory during a week is a random variable with mean 500

(i) What can be said about the probability that this week's production will be at least 1000?

(ii) If the variance of a week's production is known to equal 100, then what can be said about the probability that this week's production will be between 400 and 600? (3+3)

(b) Given two random variables X and Y, define covariance of X and Y. State any three properties of covariance. (2+3)

(c) Any item produced by a certain machine will be defective with probability 0.1, independently of any other item. What is the probability that in a sample of three items, at most one will be defective? (4)

(b) Let's say that 80% of all business start-ups in the IT industry report that they generate a profit in their first year. If a sample of 10 new IT business start-ups is selected, find the probability that exactly seven will generate a profit in their first year. (5)

(c) A deck of 52 playing cards, containing all 4 aces, is randomly divided into 4 piles of 13 cards each. Define events E_1, E_2, E_3 and E_4 as follows :

E_1={the first pile has exactly 1 ace}

E_2={the first pile has exactly 1 ace}

E_3={the first pile has exactly 1 ace}

E_4={the first pile has exactly 1 ace} (4)

(i) Normal density function of Normal Random Variable.

(ii) Cumulative distribution function of Normal Random Variable. (2+2+2)

(b) Let X and Y be two jointly continuous random variables with joint Probability Joint Function (PDF) :

$$(x,y) = \begin{cases} x + cy^2, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & otherwise \end{cases}$$

Find the constant c. (5)

(c) A fair die is rolled. Consider the following events : A={1,3,5}, B={2,3}, and C={2,3,4,5}. Find (4)

(i) P(A/B) and P(B/A)

(ii) $P(A \cup B / C)$

3. (a) Two urns contain 2 red, 3 white, and 3 red, 5 white balls, respectively. One ball is drawn at random from the first urn and transferred into the second one. A ball is then drawn from the second urn and it turns out that the ball is red. What will be the probability that the transferred ball was white? (5)

(b) Calculate E[X] when X is a Poisson random variable with parameter λ. (4)

(c) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with

probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2.

(i) Transform this process into a Markov chain.

(ii) How many states will it have after transformation?

(iii) Provide the transition probability matrix for the same. (6)

4. (a) If X is uniformly distributed over (0,10), calculate the probability that

(i) $X < 3$

(ii) $X > 7$

(iii) $1 < X < 6$ (2+2+2)