half of its investors, with the remainder switching equally to Bank A and B. Find the distribution of investors after two years. (8)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1622 **G**

Unique Paper Code : 2342011103

Name of the Paper : Mathematics for Computing

Name of the Course : **B.Sc. (H) Computer Science**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. The paper has **two** sections. **Section A** is compulsory. Each question is of **5** marks.
3. Attempt any **four** questions from **Section B**. Each question is of **15** marks.

Section – A

1. (a) State the conditions under which a system of linear equations will be consistent? Check consistency for the following system of equations : (5)

$$x_1 + x_2 + x_3 = 7$$

$$3x_1 - 2x_2 - x3 = 4$$

$$x_1 + 6x_2 + 5x_3 = 24$$

(b) Show that V_1 as $\begin{bmatrix}1\\0\\0\end{bmatrix}$ V_2 as $\begin{bmatrix}0\\1\\0\end{bmatrix}$ V_3 as $\begin{bmatrix}0\\0\\1\end{bmatrix}$ forms a basis for R^3. (5)

(c) Let Y be the set of vectors in R^4 of the form [a,0,b,0]. Prove that Y is a subspace in R^4. (5)

7. (a) Find the directional derivative of

$F(x,y,z) = 2x^2 + 3y^2 + z^2$ at P(2, 1, 3) in the direction $3\hat{i} + 4\hat{k}$. (7)

(b) Suppose that 3 banks in certain town are competing for investors. Currently bank A has 40% of the investors, Bank B has 10% and Bank C has remaining 50%. Suppose the townsfolk are tempted by various promotional campaigns to switch banks. Records show that each year Bank A keeps half of its investors, with the remainder switching equally to Bank B and C. However, Bank B keeps 2/3 of its investors, with the remainder switching equally to Bank A and C. Finally, Bank C keeps

(b) State Cayley-Hamilton theorem and verify it for the following matrix A. (8)

$$A = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$$

6. (a) Find the inverse of the following matrix using elementary row operations (7)

$$\begin{bmatrix} 1 & -4 & 1 \\ 1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$$

(b) Solve the following system of homogeneous equations by matrix method. (8)

$$5x_1 - 2x_3 = 0$$

$$-15x_1 - 16x_2 - 9x_3 = 0$$

$$10x_1 + 12x_2 + 7x_3 = 0$$

(d) Find the rank of the following matrix. (5)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$$

(e) Prove that div(curl v)=0, where v is a twice continuously differentiable vector function. (5)

(f) What do you mean by normalizing a vector? Normalize the vector [2, 3. 1, 1]. (5)

Section B

2. (a) What is a positive definite matrix? Is the following matrix positive definite? (7)

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(b) Solve the following system of linear equations using Gauss Elimination Method. (8)

$$5x_1 - 5x_2 - 15x_3 = 40$$

$$4x_1 - 2x_2 - 6x_3 = 19$$

$$3x_1 - 6x_2 - 17x_3 = 41$$

3. (a) Define orthogonality of vectors? Determine whether the vectors a and b are orthogonal or not where $a = 2\hat{i} + 6\hat{j} + \hat{k}$ and $b = 3\hat{i} - 2\hat{j} + 3\hat{k}$. (7)

(b) Diagonalize the following matrix (8)

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ -3 & 4 & 6 \end{bmatrix}$$

4. (a) Suppose L: $R^3 \rightarrow R^3$ is a linear operator and L([1, 0, 0]) = [–2, 1, 0], L([0, 1, 0]) = [3, –2, 1] and L([0, 0, 1]) = [0, –1, 3], Find L([–3, 2, 4]). Also, give formula for L([x, y, z]) for [x, y, z] $\in$ R^3. (7)

(b) Apply Gram Schmidt orthonormalization process to obtain an orthonormal basis for the subspace of R^4 generated by the vectors : (8)

$$V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad V_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} \quad V_3 = \begin{bmatrix} 0 \\ -1 \\ 2 \\ 1 \end{bmatrix}.$$

5. (a) Find gradient and curl of V for

$$V = (x^2 + y^2 + z^2)^{-3/2}\left(x\hat{i} + y\hat{j} - z\hat{k}\right). \quad (7)$$