8. (a) State and prove Newton-Cote's integration formula.

(b) Solve any **two** of the following :

(i) $u_{x+1} - 2u_x = x^2.2^x$

(ii) $u_{x+2} - 2u_x = 2^x$

(iii) $u_{x+2} - 4u_x = 5.3^x$ (7,8)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1614 **G**

Unique Paper Code : 2372012303

Name of the Paper : DSC-3, Mathematical Analysis

Name of the Course : **B.Sc. Hons. (Statistics) under UGCF-2022**

Semester : III

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **six** questions in all.

3. Question No. **1** is compulsory.

4. Attempt **five** more questions, selecting **two** from **Section-I** and three from **Section-II**.

1. Attempt any **five** parts.

(a) Give an example of

(i) A set which is not a neighborhood of any of its points.

(ii) A set which is neither closed nor open.

(iii) A set whose derived set is empty.

(b) Find the limit superior and limit inferior of $\left\langle \frac{1}{2n} \right\rangle$.

(c) Define open set. Let

$$I_n = \left]-n-\frac{1}{n}, n+\frac{1}{n}\right[, n = 1, 2, \ldots, \text{Is} \bigcap_n I_n$$

an open set? Justify your answer.

6. (a) Test for convergence the series

(i) $\frac{1}{\log 2} + \frac{1}{\log 3} + \ldots. + \frac{1}{\log n} + \ldots$

(ii) $1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{2^4} + \ldots + \frac{1}{n^n} + \ldots$

(b) Show that the series

$$1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots$$

converges absolutely $\forall x$. (8,7)

7. (a) Obtain the power series expression of $\log(1 + x)$.

(b) State Rolle's theorem and give its geometrical interpretation. (9,6)

(b) Evaluate $(2\Delta + 1)^2(x + 2)^2$, interval of difference being unity. (9,6)

Section–II

5. (a) Show that the series

$$1 + x + x^2 + \text{.........}$$

converges if $0 < x < 1$ and diverges if $x \geq 1$, oscillate finitely if $x = -1$ and oscillate infinitely if $x < -1$.

(b) Let f be defined on R by setting (8,7)

$$f(x) = |x - 2| + |x + 2|, \ \forall \ x \in R.$$

Show that f is not derivable at $x = -2$ and $x = 2$.

(d) (i) Out of total 5 cote's numbers first two are $\frac{7}{90}$ and $\frac{32}{90}$. Write the value of other three Cote's numbers.

(ii) Show that $\Delta^n f(x) = ah^n(n!)$.

(e) State Leibnitz test for alternating series. Check whether $\frac{(-1)^n}{n^3}$ is convergent or not.

(f) Compute $\Delta(x^3 + 2x^2 + 3x - 1)$.

(g) Assuming that $n^{1/n} \to 1$ as $n \to \infty$, show by applying cauchy's n^{th} root test that the series $\left(\sum_{n=1}^{\infty} n^{1/n} - 1\right)$ converges. (5×3=15)

Section–I

2. (a) Let S be a non-empty set of real numbers bounded above. Prove that a real number s is the superemum of S iff the following two conditions hold

(i) $x \le s, \ \forall \ x \in S$.

(ii) For each positive real numbers ε, there is a real number $x \in S$ such that $x > s - \varepsilon$.

(b) What is Cauchy's General Principle of convergence? Show that the sequence defined by

$$< a_n = 1 + \frac{1}{4} + \frac{1}{7} + \cdots + \frac{1}{3n-2} >,$$ does not converge.

(8,7)

3. (a) Show that the none of the following sequences $<a_n>$ is a Cauchy sequence :

(i) $a_n = (-1)^n$

(ii) $a_n = (-1)^n n$

(b) Define an open set. Show that the union of two open sets is an open set. (8,7)

4. (a) Show that for the interpolation of f(x) related to 0, α, 1. Lagrange's formula gives

$$f(x) = \left[1 - \frac{x(x-\alpha)}{1-\alpha}\right] f(0) + \frac{x(1-x)}{1-\alpha} \times \frac{f(\alpha)-f(0)}{\alpha} + \frac{x(x-\alpha)}{(1-\alpha)} \times f(1)$$

Also show that if $\alpha \to 0$, it reduces to

$$f(x) = (1 - x^2)f(0) + x(1 - x)f'(0) + x^2 f(1).$$