define two processes of moving summation. Show that both have the same correlogram.

(b) For the autoregressive series $y_{t+2} + ay_{t+1} + by_t = \varepsilon_{t+2}$, $|b| < 1$, $a^2 - 4b < 0$, show that the correlogram of order k is given by $p^k \dfrac{\sin(k\theta + \phi)}{\sin\phi}$; $k = 0, \pm1, \pm2, \ldots$

where $p = \sqrt{b}$, $\cos\theta = \dfrac{-a}{2p}$, $\tan\phi = \dfrac{1+p^2}{1-p^2}\tan\theta$. (7,8)

6. (a) Explain clearly the steps involved in Box-Jenkins approach to forecasting.

(b) For the model $(1 - B)(1 - 0.2B)y_t = (1 - 0.5B)\varepsilon_t$, classify the model as an ARIMA(p,d,q) process. Determine whether the process is stationary and invertible. Evaluate the first three ψ weights of the model when expressed as an MA(∞) model.

(c) For the SARIMA $(0,1,1) \times (1,0,0)_4$ model, obtain the 4-step-ahead forecast at time n. (5,6,4)

7. Write notes on any **two** of the following :

(a) Effect of detrending a time series

(b) Gompertz curve

(c) Exponential smoothing (7½,7½)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper :	**1156**	**C**
Unique Paper Code	: 32377905	
Name of the Paper	: Time Series Analysis	
Name of the Course	: **B.Sc. (Hons.) Statistics**	
Semester	: V	
Duration : 3 Hours	Maximum Marks : 75	

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **five** questions.
3. **All** questions carry equal marks.

1. (a) Define a time series. Describe briefly the nature of the various components of a time series.

(b) Name the characteristic movement of the time series with which you will mainly associate-

(i) an increase in the sales of soft drinks during the summer months,

(ii) a bomb blast in Delhi,

(iii) a fall in the death rate due to medical advancement,

(iv) Election of the president in India.

(c) The multiplicative model is more commonly used model as compared to the additive model in time series analysis. Give reasons. (7,4,4)

2. (a) Derive the curve of the form

$$y_t = \frac{\beta}{1+\delta e^{-\varepsilon t}}; \ \varepsilon > 0$$

Show that this is a logistic curve. Explain the method of three selected points for fitting this curve to the data regarding production in various years.

(b) What is meant by seasonal fluctuations of a time series? How do they differ from cyclic fluctuations in a time series? Describe the method of link relatives for measuring the seasonal variations, stating clearly the assumptions made. (7,8)

3. (a) In the usual notations, prove that

$$\frac{1}{h}[h]y_0 = \left[y_0 + \frac{h^2-1}{24}\delta^2 y_0\right]$$

where $\frac{1}{h}[h]$ stands for the simple average of 'h' terms.

(b) Use the above result to show that

$$\frac{1}{hkl}[h][k][l]y_0 = \left[y_0 + \frac{h^2+k^2+l^2-3}{24}\delta^2 y_0\right]$$

(c) In the above, if h = 5, k = 5, and l = 7, obtain the weights of the iterated averages when the above formula is approximated by a cubic polynomial. (5,5,5)

4. (a) Describe the method for the estimation of the variance of the random component of a time series. How is this method used for finding the degree of the trend polynomial to be fitted?

(b) Distinguish between a strict stationary process and a weak stationary process.

(c) If ε_t is a random series, show that the correlation between successive items of $\Delta^k \varepsilon_t$, for long series, is $\frac{-k}{k+1}$. Examine the case when k is large. (6,4,5)

5. (a) Let ε_t be purely random process. The relations

$$y_t = \frac{1}{2}\varepsilon_t - \frac{3}{4}\varepsilon_{t-1} - \frac{3}{8}\varepsilon_{t-2} - \frac{3}{16}\varepsilon_{t-3} - \frac{3}{32}\varepsilon_{t-4} - \cdots$$

$$y_t = \frac{1}{2}\varepsilon_t + \frac{3}{4}\varepsilon_{t-1} - \frac{3}{8}\varepsilon_{t-2} + \frac{3}{16}\varepsilon_{t-3} - \frac{3}{32}\varepsilon_{t-4} + \cdots$$