[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1050 **D**

Unique Paper Code : 2372011102

Name of the Paper : Introduction to Probability-DSC-2

Name of the Course : **B.Sc. (H) Statistics (NEP-UGCF)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **5** questions in all.
3. Question No. **1** is compulsory.
4. Attempt **4** more questions selecting any **two** questions each from **Section A** and **Section B**.

1. Attempt any **six** parts :

   (i) For any three events A, B and C defined on the sample space S such that $B \in C$ and $P(A) > 0$, show that $P(B/A) \leq P(C/A)$.

(ii) Prove that for n arbitrary independent events $A_1, A_2, \dots A_n$,

$$P\left(\bigcup_{i=1}^{n} A_i\right) = 1 - \bigcap_{i=1}^{n} P\left(A_i^c\right)$$

(iii) An urn contains 'a' white balls and 'b' black balls. A sample of two balls is drawn without replacement. Under what conditions will the probability that both the drawn balls are of the same color exceed ½.

(iv) Fifteen people sit around a circular table. What are the odds against two particular people sitting together?

(v) In a neighbourhood, 90% children were falling sick due to flu and 10% due to measles and no other disease. The probability of observing rashes for measles is 0.95 and for flu is 0.08. If a child develops rashes, find the child's probability of having flu.

(vi) The discrete random variable X has the probability distribution

x	−1	0	1	2
$P(X=x)$	p^2	p^2	$p/4$	$(4p+1)/8$

Find the value of p. Also, obtain E(X) and V(X).

(vii) Suppose Y, the grams of lead per liter of gasoline, has the density function:

$$f(y) = 12.5y - 1.25, \quad 0.1 \leq y \leq 0.5$$

Obtain cdf of Y. Calculate the probability that the next liter of gasoline has less than 0.3 grams of lead.

(viii) Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} 4x^3 & 0 \leq x \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find $P(X \leq 2/3 / X > 1/3)$. (5×6)

Section A

2. (a) For any two events A and B, prove that

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$$

(b) Let $A_1, A_2, \ldots, A_n$ be n independent events with $p(A_k) = p_k$. Further, let p be the probability that none of the events occurs. Show that $p \leq e^{-\sum_k p_k}$.

(7½×2)

3. (a) Discuss the Axiomatic approach to probability. Discuss its advantage over classical/empirical way of defining probability.

(b) A carpenter has a tool chest with two compartments, each one having a lock. He has 2 keys for each lock, and he keeps all 4 keys in the same ring. His habitual procedure in opening a compartment is to select a key at random and try it. If it fails, he selects one of the remaining three and tries, and so on. Show that the probability of success on the first, second, and third try are respectively 1/2, 1/3 and 1/6. (7½×2)

4. (a) State and prove Bayes theorem for n mutually exclusive and exhaustive events.

(b) There are three machines producing 10000, 20000 and 30000 bullets per hour respectively. These machines are known to produce 5%, 4% and 2% defective bullets respectively. One bullet is taken at random from an hour's production of the three machines. What is the probability that it is defective? If drawn bullet is defective, what is the probability that this was produced by the second machine? (7½×2)

Section B

5. (a) A special dice is prepared such that the probabilities of throwing 1,2,3,4,5 and 6 are

$$\frac{1-k}{6}, \frac{1+2k}{6}, \frac{1-k}{6}, \frac{1+k}{6}, \frac{1-2k}{6} \text{ and } \frac{1+k}{6}$$

respectively. If two such dice are thrown, find the probability of getting a sum 9.

(b) The probability that a person will die in the time interval (t_1, t_2) is given by $A\int_{t_1}^{t_2} f(t)\,dt$ where A is a constant and the function f(t) determined from long records, is

$$f(t) = \begin{cases} t^2(100-t)^2, & 0 \le t \le 100 \\ 0, & \text{elsewhere} \end{cases}$$

Find the probability that a person will die between the age 60 and 70 assuming that his age is ≥50.

(7½×2)

6. (a) The distribution of random variable X in the range (0,2) is defined by :

$$f(x)=\begin{cases} x^3, & 0<x\le 1 \\ 3(2-x)^3, & 1<x\le 2 \end{cases}$$

Calculate the mean, standard deviation and the mean deviation about mean of the distribution.

(b) An urn contains n cards marked 1 to n. Two cards are drawn at a time. Find the mathematical expectation of the product of the numbers on the cards. (7½×2)

7. (a) State and prove multiplication theorem of expectation. If two unbiased dice are thrown, then find the expected values of the sum of numbers of points on them.

(b) Let $f(x)=\begin{cases} \frac{1}{2}, & -1<x<1 \\ 0, & \text{elsewhere} \end{cases}$

be the pdf of the random variable X. Find the distribution function and pdf of $Y = X^2$.

(7½×2)