(b) Prove duplication formula

$$\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2n-1}}\Gamma(2n).$$

8. (a) Evaluate $\lim_{x\to 0}\left(\frac{1}{x^2}-\tan^2 x\right)$.

(b) Form partial differential equation by eliminating a and b from $z = (x^2 + a)(y^2 + b)$.

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1071 **D**

Unique Paper Code : 2372011103

Name of the Paper : Calculus

Name of the Course : **B.Sc. (Hons) Statistics (NEP)**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. **All** questions/parts carry equal marks.

1. (a) Evaluate $\lim_{x\to\pi/2}\frac{a^{\sin x}-a}{\log\sin x}$.

(b) Solve $\frac{1}{y}\frac{dy}{dx}+\frac{x}{1-x^2}=\frac{x}{\sqrt{y}}$.

2. (a) If $u = e^x (x \cos y - y \sin y)$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 .$$

(b) Using differentiation under the integral sign, evaluate $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)} dx$.

3. (a) Solve $(D^2 - 2D + 1)y = xe^x \sin x$.

(b) If $y = x(x+1)\log(x+1)^3$, then prove that

$$y_n = \frac{3(-1)^{n-1}(n-3)!(2x+n)}{(x+1)^{n-1}}, \quad n \geq 3 .$$

4. (a) If

$$f(x) = \begin{cases} \left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{1/x}, & x \neq 0, \\ k, & x = 0. \end{cases}$$

then for what value of k, f(x) is continuous at x = 0?

(b) Verify Euler's theorem for $z = \frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{5}} + y^{\frac{1}{5}}}$.

5. (a) Solve partial differential equation $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.

(b) Show that $\left(\int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}}\right)\left(\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta\right) = \pi$.

6. (a) Change the order of integration

$$\int_0^{2a} \int_{\sqrt{2ax - x^2}}^{\sqrt{2ax}} f(x, y)\, dy\, dx$$

and evaluate it for f(x, y) = 1.

(b) Solve differential equation $x\frac{dy}{dx} + y = y^2 \log x$.

7. (a) Solve the differential equation :

$$(2x+3)^2 \frac{d^2y}{dx^2} + 2(2x+3)\frac{dy}{dx} + 12y = 6x$$