Unique paper code : 32371109

Name of the paper : Calculus

Name of the course : B.Sc.(Hons) Statistics (CBCS)

Semester : I

Duration : 3 Hours

Max. Marks : 75 Marks

Instructions for candidates
Attempt four questions in all. All questions carry equal marks.

1(a) If $\theta = t^n e^{-\frac{r^2}{4t}}$, find value of n which will make $\dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial\theta}{\partial r}\right) = \dfrac{\partial\theta}{\partial t}$.

(b)   Evaluate $\lim\limits_{n \to \infty} \sum\limits_{r=1}^{3n} \dfrac{n^2}{(3n+r)^3}$.

2 (a)   If $f\left(\dfrac{y}{x}\right) = \dfrac{1}{xy}$, show that $\dfrac{f'\left(\dfrac{y}{x}\right)}{f\left(\dfrac{y}{x}\right)} = \dfrac{x\left(y + x\dfrac{dy}{dx}\right)}{y\left(y - x\dfrac{dy}{dx}\right)}$.

(b)   Determine maximum value of $u = \sin x \sin y \sin(x+y)$.

3 (a)   Define Beta and Gamma integral and establish relation between them.

(b)   Change the order of integration in the double integral $\displaystyle\int\limits_{0}^{1}\int\limits_{x^2}^{2-x} x\, y\, dy\, dx$ and hence evaluate it.

4 (a)   Solve the differential equations: $\left(9D^2 + 3D - 1\right)y = e^x \cos 2x$, where $D = \dfrac{d}{dx}$.

(b)   Solve partial differential equation: $z^2\left(p^2 + y^2 q^2\right) = 1$.

5 (a)   Solve Lagrange's partial differential equation: $\dfrac{y-z}{x}p + \dfrac{z-x}{y}q = \dfrac{x-y}{z}$,

(b)   Prove that: $\dfrac{B(p,q+1)}{q} = \dfrac{B(p+1,q)}{p} = \dfrac{B(p,q)}{p+q}$ where $p > 0$, $q > 0$.

6 (a)   Solve the differential equations: $(1+4x)^2\dfrac{d^2y}{dx^2} - 6(1+4x)\dfrac{dy}{dx} + 16y = 9(1+4x)^4$.

(b)   Solve partial differential equation: $\dfrac{\partial^2 z}{\partial x^2} - 9\dfrac{\partial^2 z}{\partial x\,\partial y} + 20\dfrac{\partial^2 z}{\partial y^2} = e^{2x+3y}$.