$Y_2 = \frac{X_1}{X_1 + X_2}$, and hence show that they are independent. (7,8)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1208 **F**

Unique Paper Code : 2372011201

Name of the Paper : Theory of Probability Distributions DSC

Name of the Course : **B.Sc. (Hons.) Statistics – DSC-3**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all selecting **three** questions from each section.
3. Use of a non-programmable scientific calculator is allowed.

Section A

1. (a) Define cumulants and cumulants generating function. Obtain first four cumulants (κ_r) in terms of moments about mean (μ_r).

(b) Let X be a random variable having probability density function (p.d.f) as

$$f(x)=\frac{1}{2\theta}\exp\left(-\frac{|x-\theta|}{\theta}\right); \quad -\infty<x<\infty.$$

Find moment generating function (m.g.f.) of X. Hence, find E(X) and V(X). (7,8)

2. (a) Define characteristic function $\Phi_X(t)$ of a random variable X. Show that

(i) $\Phi_X(t)$ and $\Phi_X(-t)$ are conjugate functions,

(ii) $\Phi_X(t)$ is real valued and even function if $f(x) = f(-x)$, where f(x) is the probability density function of X.

8. (a) Let X_1 and X_2 are independent Normal variates such that

$$X_1 \sim N(1, 9) \text{ and}$$
$$X_2 \sim N(2, 16).$$

Let Z be a random variable such that $Z = X_1 - X_2$, then find

(i) probability density function of Z,

(ii) mean, variance and median of Z, and

(iii) $P(Z + 1 \le 0)$.

(b) Let X_1 and X_2 be two independent random variables having same p.d.f., given as

$$f(x)=\begin{cases}\exp(-x), & 0<x<\infty\\ 0, & \text{otherwise}\end{cases}$$

Find the joint p.d.f. of $Y_1 = X_1 + X_2$ and

$$\mu_{r+1} = pq\left(nr\mu_{r-1} + \frac{d\mu_r}{dp}\right), \quad r = 1, 2, \ldots\ldots$$

Hence obtain μ_2 and μ_3. (7,8)

7. (a) Show that for uniform distribution:

$$f(x) = \frac{1}{2a}; \quad -a < x < a,$$

Find m.g.f. about origin. Also show that moments of even order are given by :

$$\mu_{2n} = \frac{a^{2n}}{(2n+1)}, \quad n = 0, 1, 2, \ldots..$$

(b) Let random variable X follow normal distribution with mean μ and variance σ^2. Find r^{th} moment about mean (μ_r) and deduce that

$$\mu_{2n+1} = 0, \qquad n = 0,1,2, \ldots..$$

$$\mu_{2n} = 1.3.5 \ldots.(2n-1)\sigma^{2n}, \qquad n = 0,1,2, \ldots..$$

(7,8)

(b) Let X be a random variable with r^{th} moment about origin as given below

$$\mu'_r = (r+1)!\ 2^r.$$

Find (i) Moment generating function of X,

(ii) Cumulant generating function of X, and

(iii) Characteristic function of X. (7,8)

3. (a) Explain the following notations :–

(i) $R_{1.23}$

(ii) $\sigma^2_{1.23}$

(iii) $r_{12.3}$

Also, write their expressions in terms of r_{12}, r_{13} and r_{23}.

(b) Two random variables X and Y have the following joint probability density function:

$$f(x,y)=\begin{cases}2-x-y; & 0\le x\le 1,\quad 0\le y\le 1\\ 0\;; & \text{otherwise}\end{cases}$$

Find (i) Marginal p.d.f. of X,

(ii) Conditional p.d.f. of $(Y|X = x)$,

(iii) $E(Y|X = x)$,

(iv) $E\left(XY\middle|X=\frac{1}{2}\right)$. (7,8)

4. (a) Derive the equation of the plane of regressions of X_1 on X_2 and X_3, assuming

$$\bar{X}_1=\bar{X}_2=\bar{X}_3=0\,.$$

(b) If r_{12} and r_{13} are given, show that r_{23} must lie in the range:

$$r_{12}r_{13} \pm (1 - r_{12}^2 - r_{13}^2 + r_{12}^2\, r_{13}^2)^{1/2}.$$

If $r_{12} = k$ and $r_{13} = -k$, show that r_{23} will lie between -1 and $1 - 2k^2$. (7,8)

Section B

5. (a) If X is a Poisson variate with mean λ, show that

(i) $E(X^2) = \lambda\, E(X + 1)$.

(ii) If $\lambda = 1$, show that $E|X - 1| = \frac{2}{e}$ s. d. (X),

where, s.d. is the standard deviation of X.

(b) Let X is a Poisson variate with mean θ. Find m.g.f of X and m.g.f of $Y = 2X - 1$. Hence, find mean and variance of Y. (7,8)

6. (a) Find mode of $B(n, p)$. Hence find mode if $n = 7$ and $p=\frac{1}{3}$.

(b) If $X \sim B(n, p)$ and μ_r is the r^{th} moment about mean, prove that

P.T.O.