[This question paper contains 6 printed pages.]

Your Roll No. 21020568015

Sr. No. of Question Paper : 4534 E

Unique Paper Code : 32371401

Name of the Paper : Statistical Inference

Name of the Course : B.Sc. (H) Statistics

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **six** questions in all, selecting **three** questions from each section.

SECTION 1

1. (a) State and prove the Invariance property of consistent estimator. Give an example to show that

   (i) The estimator is both consistent and unbiased

(ii) The estimator is consistent but not unbiased.

(b) Define unbiasedness property of an estimator. Find the only unbiased estimator based on a single observation from $P(\theta)$ for $\gamma(\theta) = e^{-(k+1)\theta}$. Comment on the result. (6.5,6)

2. (a) Let X_1, X_2 be a random sample of size 2 from a distribution having p.d.f.

$$f(x,\theta) = \frac{1}{\theta}e^{\frac{x}{\theta}};\ x > 0,\ \theta > 0.$$

(i) Find a sufficient statistic for θ.

(ii) Find joint p.d.f. of $Y_1 = X_1 + X_2$ and $Y_2 = X_2$. Show that Y_2 is an unbiased estimator of θ with variance θ^2.

(iii) Find $E[Y_2 \mid Y_1 = y_1] = \phi(y_1)$, $E[\phi(Y_1)]$ and $Var[\phi(Y_1)]$.

(iv) Interpret the result.

(b) Let T_1 and T_2 be two unbiased estimators of $\gamma(\theta)$ with efficiencies e_1 and e_2 respectively. ρ_θ is the correlation coefficient between them. Show that

$$\sqrt{e_1e_2} - \sqrt{(1-e_1)(1-e_2)} \le \rho_\theta \le \sqrt{e_1e_2} + \sqrt{(1-e_1)(1-e_2)}$$

Discuss the case when $e_1 = 1$ and $e_2 = e$. (6.5,6)

3. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0,\theta)$ population. Show that the largest order statistic $X_{(n)}$ is a complete sufficient statistic for θ. Using Lehman Schetf'e theorem, find MVUE of θ.

(b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a population defined by

$$f(x,\theta) = \theta(1-\theta)^{x-1};\ x = 1,2,\ldots$$

Estimate θ by the

(i) Method of moments

(ii) Method of MLE. (6,6.5)

4. (a) State Cramer-Rao Inequality. When does the inequality become equality in C-R inequality? Obtain Minimum variance bound estimator of θ based on a random sample of size n from $N(\mu, \theta)$ distribution, where μ is known.

(b) Describe the method of maximum likelihood estimation. Let $X_1, X_2, \ldots, X_n$ be a random sample

from $f(x, \theta) = 1;\ \theta - \frac{1}{2} \le x \le \theta + \frac{1}{2}, -\infty < \theta < \infty$.

Obtain MLE of θ and comment on the result.

(6.5,6)

SECTION 2

5. (a) Let X be a $B(n,\theta)$ random variable and prior distribution of θ is Beta distribution with parameters α and β. Find the posterior distribution of θ given $X = x$. Also find Bayes' estimator of θ under squared error loss function (SELF).

(b) Define MPCR and UMPCR. State Neyman-Pearson lemma for constructing the BCR. Let $X_1, X_2, \ldots, X_n$ be a random sample from $f(x,\theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$. Find MP test of size α based on chi-square statistic for testing $H_0: \theta = \theta_0$ against $H_1: \theta = \theta_1 (> \theta_0)$. Also obtain the power of the test. (6,6.5)

6. (a) Develop LR test for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$, when a sample of size n is drawn from $N(\theta,\sigma^2)$, where σ^2 is known.

(b) Describe a method of constructing confidence intervals. Using first order statistic $X_{(1)}$, find $100(1-\alpha)\%$ confidence interval for θ based on a random sample of size n from ap.d.f. $f(x,\theta) = e^{-(x-\theta)};\ \theta \le x < \infty, -\infty < \theta < \infty$. (6.5,6)

7. (a) Find $100(1-\alpha)\%$ confidence interval for binomial proportion θ based on a random sample of size n for large samples.

(b) Let X_1, X_2 be a random sample of size 2 from a distribution having p.d.f $f(x, \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $\theta > 0$. Find the size of the test for testing $H_0: \theta = 1$ against $H_1: \theta = 2$ and having critical region as

$$W = \left\{(x_1, x_2) : x_1 x_2 \ge \frac{3}{4}\right\}.$$

(c) Find the minimum mean square error estimator of the form aS^2 for σ^2 based on a random sample of size n from $N(\theta,\sigma^2)$, θ is known. (4,4,4.5)

8. Write a short note on **any three** of the following :

(i) Sufficient conditions for consistent estimator

(ii) Factorisation theorem

(iii) Method of minimum chi-square

(iv) Blackwellisation process in estimation theory

(4.5,4,4)