[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4552 **E**

Unique Paper Code : 32371208

Name of the Paper : Probability and Probability Distributions

Name of the Course : **B.Sc. (H) STATISTICS under CBCS**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **6** questions in all.

3. Question No. **1** is compulsory.

4. Attempt **5** more questions selecting at least **two** questions from each section.

1. (a) Fill in the blanks :

   (i) A discrete distribution for which mean = variance is ________ .

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(ii) Var(X) of a degenerate random variable is ________ .

(iii) If X and Y are independent binomial variates B(6,1/4) and B(5,1/4) respectively, then $P(X + Y = 5) =$ ________ .

(iv) If $X_1, X_2, \ldots, X_n$ are n independent observations from a standard Cauchy distribution, then the distribution of $\bar{X}$ is ________ .

(v) Lower and Upper limits of multiple correlation coefficient $R_{1.23}$ are ________ and ________ respectively.

(b) Can $Y = 5 + 2.8X$ and $X = 3 - 0.5Y$ be the estimated regression equations of Y on X and X on Y respectively? Explain your answer.

(c) If X and Y are independent variates such that $X \sim N(2,4)$ and $Y \sim N(3,5)$, then find the standard deviation and β_1 of $(X + 5Y)$.

(d) If X has an exponential distribution with mean λ, find $P(X < 1 \mid X < 2)$.

(e) A random variable X has p.m.f

$$p(x) = \frac{1}{2^x}; \quad x = 1, 2, 3, \ldots$$

Find its m.g.f and hence mean.

(f) State the relationship between hypergeometric distribution and binomial distribution.

(1×5,2,2,2,2,2)

Section I

2. (a) Find mode of Poisson distribution. Hence find the mode if parameter of the distribution is 5.

(b) Let X be a random variable with pdf

$f(x) = kxe^{-\lambda x}dx; \quad 0 < x < \infty, \lambda > 0,$ where k is a constant

Find mean, variance, β_1 and β_2 for the distribution. Also comment on the symmetry of the distribution.

(6,6)

3. (a) Suppose X is a non-negative integral valued random variable. Show that the distribution of X is geometric iff it has memoryless property.

(b) Define random variable. Let X be a random variable with p.d.f:

$$f(x) = ke^{-|x|} \quad -\infty < x < \infty$$

where k is a constant. Find the p.d.f of $Y = X^2$.

(6,6)

4. (a) Ten tickets are drawn with replacement from a bag containing 100 tickets numbered 1, 2, ..., 100. Find the mathematical expectation of :

(i) sum of numbers on the ticket drawn.

(ii) product of numbers on the tickets drawn.

(b) Define moment generating function of the random variable X. How will you get moments from moment generating function. (6,6)

Section II

5. (a) Explain the concepts of multiple and partial correlation coefficients. Show that the multiple correlation coefficient $R_{1.23}$, in usual notations is given by :

$$R_{1.23}^2 = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}}{1 - r_{23}^2}$$

(b) If X and Y are two random variables with variances σ_X^2 and σ_Y^2 respectively and r is the correlation coefficient between them. If $U = X + kY$, and $V = X + (\sigma_X/\sigma_Y)Y$, find the value of k so that U and V are uncorrelated.

(6,6)

6. (a) What is Legendre's principle of least squares? Derive the normal equations for fitting of a curve of the type $XY = aX^2 + b$ to a set of n points (X_i, Y_i), $i = 1,2,\ldots, n$.

(b) Define Normal distribution. Prove that a linear combination of independent normal variates is also a normal variate. (6,6)

7. (a) Find the mean value of positive square root of a one parameter gamma variate and hence evaluate the mean deviation of normal variate from its mean.

(b) Define exponential distribution and find its moment generating function and hence compute its arithmetic mean and variance. (6,6)

8. (a) Define expectation. Prove that if $E(X^r)$ exists, then $E(X^s)$ exists for all $1 \leq s \leq r$.

(b) Define the characteristic function of a random variable X. Show that the characteristic function of the sum of two independent random variables is equal to the product of their characteristic functions. Is the converse true? Justify your answer. (6,6)