(b) Describe the four scales of measurement that have the most practical implications for researchers.

(c) Describe the advantages and disadvantages of non-parametric statistical tests. (4,4,4)

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4796 **E**

Unique Paper Code : 32371602

Name of the Paper : Multivariate Analysis and Non-parametric Methods

Name of the Course : **B.Sc. (H) Statistics under CBCS (LOCF)**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Question **1** is compulsory.

3. Attempt **five** more questions selecting three questions from **Section A** and **two** questions from **Section B**.

4. Use of a non programmable scientific calculator is allowed.

1. Attempt all parts :

(a) Fill in the blanks :

(i) Factor analysis is a ________ technique.

(ii) SPRT was developed by ________ and is used for testing ________ hypothesis against ________ hypothesis.

(iii) If all the p random variables are independent then the variance-covariance matrix is ________ .

(iv) The limits of multiple correlation coefficients are ________ .

(b) The first two moments of the null distribution of the number of runs r, for two independent samples of sizes m and n are given by ______ and ______ .

(c) If (X, Y) ~ BVN(0,0,1,1 ρ), find the distribution of $Z=\frac{X}{Y}$.

(d) (i) The critical region for the level α (equal tailed) test for testing H_0: $\mu_e = \mu_{e0}$ against H_1: $\mu_e \neq \mu_{e0}$ in case of a random sample of

SECTION B

6. (a) Determine the constants A and B in terms of the error probabilities α and β in the SPRT.

(b) Let the random variables X_1, X_2,... be independent and identically distributed as N(μ, 1) with unknown μ. Determine SPRT for testing H_0: $\mu = \mu_0$ against H_1: $\mu = \mu_1 (> \mu_0)$. (6,6)

7. (a) Describe a suitable non parametric test for testing the null hypothesis that the k independent samples come from the same population or from identical populations with the same median. Also explain the case when there are tied observations in the samples.

(b) Describe a suitable non-parametric test for testing whether a sequence of observations in a sample can be considered to be random or not. (8,4)

8. (a) Explain the non parametric median test for two independent samples.

size n having r and s as number of positive and negative signs respectively, is given by ______ and ______.

(ii) Define multivariate normal distribution for p-random variables. Also, define mean vector and dispersion matrix. (6,2,3,2×2)

SECTION A

2. (a) If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then prove that

$$P(X > \mu_1, Y > \mu_2) = \frac{1}{4} + \frac{\sin^{-1}(\rho)}{2\pi}.$$

(b) If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, then find the correlation coefficient between e^X and e^Y.

(6,6)

3. (a) If $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ and $\underline{Y} = C\underline{X}$, where C is a non-singular matrix, then show that $\underline{Y} \sim N_p(C\underline{\mu}, C\Sigma C')$.

(b) If $(X, Y) \sim BVN(0,0,1,1,\rho)$, then show that moments obey the recurrence relation

$\mu_{r,s} = (r+s-1)\ \rho\mu_{r-1,s-1} + (r-1)(s-1)(1-\rho^2)\ \mu_{r-2,s-2}$.

Hence or otherwise, show that $\mu_{3,1} = 3\rho$.

(6,6)

4. (a) Let $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$. Partition $\underline{X}$ into $\underline{X}^{(1)} = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_q \end{pmatrix}$ and $\underline{X}^{(2)} = \begin{pmatrix} X_{q+1} \\ X_{q+2} \\ \vdots \\ X_p \end{pmatrix}$, consequently, mean vector and variance covariance matrix is partitioned as $\mu = \begin{pmatrix} \underline{\mu}^{(1)} \\ \underline{\mu}^{(2)} \end{pmatrix}$ and $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$. Obtain the marginal distributions of $\underline{X}^{(1)}$ and $\underline{X}^{(2)}$.

(b) Explain the concept of multiple and partial correlation coefficients with illustrations. How do we estimate the value of X_3 for given values of X_2 and X_1? (6,6)

5. (a) Describe the method of principal component analysis. What are its uses?

(b) If $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$, then show that the quadratic form $Q = (\underline{X} - \underline{\mu})'\Sigma^{-1}(\underline{X} - \underline{\mu})$ follows chi-square distribution with p d.f. (6,6)