(b) Define Exponential distribution and obtain its Moment Generating Function. Hence find its mean and variance. (6,6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6113 **E**

Unique Paper Code : 32375201

Name of the Paper : GE-II: Introductory Probability

Name of the Course : **B.Sc. (H) Statistics under (LOCF) Repeat**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. **Section A** is compulsory.

3. Attempt any **five** questions, selecting at least **two** questions from each of the **Sections B** and **C**.

4. Use of simple calculators is allowed.

Section A

1. Answer the following :

(a) If P(A) = 0.3, P(B) = 0.2 and $P(A \cup B) = 0.4$ then value of $P(\bar{A} \cap \bar{B})$ is? (1)

(b) Let determine whether f(x) can serve as the probability distribution of a random variable. (1)

(c) If E(X) = 10, E(Y) = 8, Var(X) = 20, Var(Y) = 15, then is? (1)

(d) If X follows exponential distribution with parameter 4, then Var(X) is? (1)

(e) If are probabilities of three mutually exclusive and exhaustive events then the value of p? (1)

(f) If = ------- (2)

7. (a) A manufacturer, who produces medicine bottles, finds that 0.1% of the bottles are defective. The bottles are packed in boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles. Using Poisson distribution, find how many boxes will contains :

(i) no defectives and

(ii) at least two defectives.

(b) Define Uniform distribution. Obtain moments of the uniform distribution, and hence find its mean and variance. (6,6)

8. (a) If two percent of the books bound at a certain bindery have defective bindings. Use the Poisson approximation to the binomial distribution to determine the probability that five of 400 books bound by this bindery will defective bindings.

Section C

5. (a) State Chebyshev's inequality and prove it when X is a continuous random variable.

(b) Use Chebyshev's inequality to determine how many times a fair coin must be tossed in order that the probability will be atleast 0.90 that the ratio of the observed number of heads to the number of tosses will lie between 0.4 and 0.6. (6,6)

6. (a) Define Negative Binomial distribution. Give an example in which it occurs? Show that negative binomial distribution may be regarded as the generalization of geometric distribution.

(b) If the probability is 0.75 that an applicant for a driver's license will pass the road test on any given try, what is the probability that an applicant will finally pass the test on the fourth try?

(7,5)

(g) Name a discrete and a continuous distribution for which variance mean. (2)

(h) If, find E(X) and Var(X). (2)

(i) If $\mu_X = 10$, $\mu_Y = 20$, $\sigma_X^2 = 25$, $\sigma_Y^2 = 36$ and Cov(X,Y) = 100, then Var(4X – 3Y) is? (2)

(j) State the conditions under which Poisson distribution is a limiting case of the binomial distribution. (2)

Section B

2. (a) Two socks are selected at random and are removed in succession from a drawer containing five brown socks and three green socks. If random variable W represents number of brown socks selected then find probability distribution of W and E(W).

(b) Suppose that a product is produced in three factories X, Y, and Z. It is known that factory X produces thrice as many items as factory Y, and that factories Y and Z produce the same number of items. Assume that it is known that 3 percent of the items produced by each of the factories X and Z are defective while 5 percent of those manufactured by factory Y are defective. All the items produced in the three factories are stocked, and an item of product is selected at random.

(i) What is the probability that this item is defective?

(ii) If an item is selected at random is found to be defective, what is the probability that it was produced by factory X, Y and Z respectively? (6,6)

3. (a) In four tosses of a coin, let X be the number of heads. Tabulate all the possible outcomes with the corresponding values of X. Derive the probability distribution and hence calculate the expected value of X.

(b) Suppose that the time in minutes that a person has to wait at a certain bus stop for a bus is found to be a random phenomenon, with a probability density function given by

(i) Obtain F(x), the c.d.f. of X.

(ii) What is the probability that a person will have to wait more than two minutes?

(iii) Obtain mean of X. (6,6)

4. (a) Find mean and variance of random variable X whose probability density function is :

$$f(x)=\begin{cases}\dfrac{4}{\pi(1+x)^2}, & 0<x<1\\ 0, & \text{otherwise.}\end{cases}$$

(b) State the three properties of moment generating function. If a random variable X has the probability distribution for x = 0, 1, 2 and 3, find the moment generating function of the random variable X and use it to determine. (5,7)