Strike price	Put price	Call price
98	0.4394	14.3782
100	0.6975	12.7575

(b) An investor longs a 3-months maturity call option with K = 220 and shorts a half-year call with K = 220. Currently S_0 = 220, risk-free rate r = 0.06 (continuously compounded), and σ = 0.25. The stock pays no dividend. Calculate Δ, Γ and θ of the portfolio.

Table of Φ(z) or CDF of N[0,1] values

z =	0.03	0.05	0.06	0.08	0.10	0.18	0.20	0.25	0.30
Φ(z) =	0.5119	0.5199	0.5239	0.5318	0.5398	0.5714	0.5792	0.5987	0.6179

(6½,6)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4892 **E**

Unique Paper Code : 32377911

Name of the Paper : DSE-4: Financial Statistics

Name of the Course : **B.Sc. (H) Statistics under CBCS**

Semester : VI (LOCF)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **Five** questions in all.

2. **Section A** is compulsory.

3. Attempt **Two** questions from each of **Sections B** and **C**.

4. Use of non-programmable Scientific calculator is permitted.

Section-A

1. Attempt any **five** parts. **All** parts carry equal marks.

(a) What is the value of a 5-year 7.4% coupon bond selling to yield 5.6% per annum (nominal converted semi-annually) assuming the coupon payments are made semi-annually and the bond is redeemed at par?

(b) Suppose that the yields to maturity on the zero-coupon bonds of various maturities with continuous compounding are as follows:

Maturity (years)	Rate (% per annum)
1	2.0
2	3.0
3	3.7
4	4.2

Calculate forward interest rates for the second, third, and fourth years.

6. (a) Consider the process

$$dS_t = \mu S_t dt + \sigma dW_t;\ S_0 > 0$$

Investigate dynamics of the Forward price process $F_t = S_t e^{r(T-t)}$; $S_0 > 0$. Identify the distribution of F.

(b) In a one-step binomial tree model, it is assumed that the initial share price of 260 will either increase to 285 or decrease to 250 at the end of one year. Calculate the price of each of the following options assuming continuous rate of interest 5% and that no dividends are payable.

(i) a one-year European call option with a strike price of 275

(ii) a one year European put option with a strike price of 275.

Verify numerically that the put-call parity relationship holds in this case. (6½,6)

7. (a) A stock pays at a rate proportional to its price the dividend yield is 2%. You are given the following option prices for European puts and calls with time to expiry 2 years. Calculate the current prices of the stock.

(b) Consider a European call option on a stock with current spot price $S_0 = 20$, dividend D = ₹2, exercise price K = 18 and time to maturity 6 months. The annual risk-free rate is r = 10%. What are the upper and lower limits of the price of the call options? (6½,6)

Section-C

5. (a) For a two-period binomial model, it is given that each period is of one-year length. The current price of a non-dividend paying stock is 20. The up- movement u =1.284 and the down movement d = 0.8607. The continuously compounded rate of interest is 5% per year. For a strike price of K = 22, calculate the price of an American put option.

(b) Let $\{Y_t;\ t \geq 0\}$ be an Itô-process given by $dX_t = \mu\{t, X_t)dt + \sigma(t,X_t)dW_t$. Further let $Y_t = g(t, X_t)$ is a differentiable function of t and X_t. Then show that the SDE for $\{Y_t;\ t \geq 0\}$ is given by

$$dY_t = \left(\frac{\partial g(t,X_t)}{\partial X}\mu(t,X_t) + \frac{1}{2}\frac{\partial^2 g(t,X_t)}{\partial X^2}\sigma^2(t,X_t) + \frac{\partial g(t,X_t)}{\partial t}\right)dt + \frac{\partial g(t,X_t)}{\partial X}\sigma(t,X_t)dW_t$$

(6½,6)

(c) A company has a project that will require an initial investment of $1,000,000. The project will produce cash inflows of $500,000 the first year, $700,000 the second year, and $1,000,000 the third year. The company's required rate of return is 10%. What is the NPV of the project?

(d) The correlation (ρ) between two assets 'A' and 'B' is 0.1. The expected returns and standard deviations of returns are given in the following table :

Asset	Return (%)	Standard deviation (%)
A	10	15
B	18	30

(i) Find the proportions α and (1 – α) of 'A' and 'B' respectively that defines a portfolio consisting of them such that it has minimum variance.

(ii) Find the expected return and the standard deviation of this portfolio.

(e) Let W_t be a standard Wiener process. Derive the value of $E\left(\int_0^t W_s^3 dW_s\right)$.

(f) Consider a forward contract on a non-dividend paying share for which the stock price process is an Itô-process. If the risk-free rate of interest is 0%, then find the SDE for the contract F_t maturing at time T.

(g) Let W_t be a standard Wiener process. Define $X_t = e^{\lambda W_t - \frac{1}{2}\lambda^2 t}$ where λ is a constant. Show that X_t is a martingale with respect to F_t, the filtration associated with W_t.

(h) Obtain 'delta' of a European put option.

(5×5=25)

Section-B

2. (a) Explain the terms Arbitrage and Arbitrageurs with the help of an example.

(b) Explain 'The Comparison Principle' with an example in the context of financial markets.

(c) Consider two five- year bonds: one has a coupon of 9% and sells for ₹101.00; and the other has a coupon of 7% and sells for ₹93.20. Find the price of a 5-year zero-coupon bond. (4½,4,4)

3. (a) Suppose that put options on a stock with strike prices \$30 and \$35 cost \$4 and \$7, respectively. How can the options be used to create (a) a bull spread and (b) a bear spread? Construct a table that shows the profit and payoff for both spreads.

(b) For two European puts with the same maturity date T and delivery prices $K_1 \leq K_2$, show that at time $t < T$:

$$0 \leq P_{K_2,T}(S_t, \tau) - P_{K_1,T}(S_t, \tau) \leq (K_2 - K_1)e^{-r\tau}$$

where τ denotes the time to maturity and r is the interest rate. (6½,6)

4. (a) Construct the payoff table along with the payoff graph for the following two alternative strategies that uses options :

Strategy 1: consists of buying a European call option with a strike price ₹110 and selling a call option on the same stock with higher strike price of ₹120. Both call options have same expiration date.

Strategy 2: consisting of a long call with delivery price ₹40 and a long put with delivery price ₹40; and same expiration date.