(b) Prove that if G is a generalised inverse of $X'X$, then

(i) G' is also a generalised inverse of $X'X$.

(ii) $G X'$ is a generalised inverse of X.

(iii) $XG X'$ is invariant to G.

(iv) $XG X'$ is symmetric, whether G is, or not.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1246 **F**

Unique Paper Code : 2372011203

Name of the Paper : Algebra of Statistics

Name of the Course : **B.Sc. (H) Statistics – DSC-5**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **six** questions in all, selecting **three** from each Section.

3. **All** questions/parts carry equal marks.

4. Use of non-programmable scientific calculators is allowed.

Section A

1. (a) Define Involutory, Hermitian and Unitary matrices. Show that the matrix A is a Nilpotent matrix of index 2, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ -1 & -2 & -3 \end{pmatrix}$$

(b) If $A = (a_{ij})$ is any m x n matrix, then prove that

(i) $tr(AA') \geq 0$

(ii) If A is a symmetric matrix of order n then

$$tr(A^2) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^2$$

2. (a) Suppose that a $(p + q) \times (p + q)$, M matrix can be partitioned into four sub-matrices as

7. (a) What is meant by Spectral Decomposition of a symmetric matrix? If $A_i(i = 1,2,......s)$ be 's' real symmetric matrices, each of order n, such that $\Sigma_{i=1}^{s} A_i = I$, where I is the unit matrix of order n, and if r_i be the rank of A_i such that $\Sigma_{i=1}^{s} r_i = n$, show that each of the non-zero characteristic roots of A_i is +1.

(b) How can we find the inverse of a non-singular matrix by using elementary transformations? Find the matrix F which, when pre-multiplied by A, interchanges columns 1 and 3 and multiplies column 2 by 2, where

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 1 & 5 & 4 \end{pmatrix}$$

8. (a) Using Gram- Schmidt Orthogonalization process, construct an orthonormal basis from

$x_i = [2\ 1\ 2]$, $x_2 = [4\ 1\ 0]$, $x_3 = [3\ 1\ -1\ ]$.

(b) Reduce the quadratic form

$$X'AX = 6x_1^2 + 3x_2^2 + 14x_3^2 + 4x_2x_3 + 18x_3x_1 + 4x_1x_2$$

into canonical form and hence determine its rank and index.

6. (a) State and prove Cayley Hamilton theorem and use it to find A^{-2}, where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(b) Reduce the given matrix B to the normal form and hence obtain its rank,

$$B = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & -4 & 7 \\ -1 & -2 & -1 & 1 \end{bmatrix}$$

$$M = \begin{pmatrix} A & B \\ B' & D \end{pmatrix}$$

where A and D are symmetric matrices of dimensions $p \times p$ and $q \times q$ respectively. Show that

$$M^{-1} = \begin{pmatrix} A^{-1} + FE^{-1}F' & -FE^{-1} \\ -E^{-1}F' & E^{-1} \end{pmatrix}$$

where $E = D - B'A^{-1}B$, $F = A^{-1}B$ and such that the inverses which occur in the expression exist.

(b) Express

$$A = \begin{vmatrix} (1+ax)^2 & (1+ay)^2 & (1+az)^2 \\ (1+bx)^2 & (1+by)^2 & (1+bz)^2 \\ (1+cx)^2 & (1+cy)^2 & (1+cz)^2 \end{vmatrix}$$

as a product of two determinants and solve it.

3. (a) Reduce the following matrix to Row-reduced Echelon form and mark the pivots.

$$A = \begin{pmatrix} 0 & 2 & 2 & -1 & 3 \\ 1 & 2 & 5 & 0 & 7 \\ 1 & 4 & 7 & 0 & 13 \\ 3 & -2 & 7 & 2 & 3 \end{pmatrix}$$

(b) If A is a non-singular matrix of order n then show that :

(i) $|\text{adj } A| = |A|^{n-1}$

(ii) $\text{adj}(\text{adj } A) = |A|^{n-2}.\ A$

(iii) $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

4. (a) Show that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix}$$

Further show that its value comes out to be $(y-z)\ (z-x)\ (x-)(yz + zx + xy)$.

(b) Examine the consistency of the following system of equations :

$$x_1 + 2x_2 + 4x_3 + x_4 = 4$$

$$2x_1 - x_3 - 3x_4 = 4$$

$$x_1 - 2x_2 - x_3 = 0$$

$$3x_1 + x_2 - x_3 - 5x_4 = 5$$

Section B

5. (a) Define Vector Spaces and Subspaces. Prove that the vectors (1 2 4), (1 0 0), (0 1 0) and (0 0 1) are linearly dependent.