moment generating function of $Z = \dfrac{X - n\theta}{\sqrt{n\theta(1-\theta)}}$ approaches that of the standard normal distribution when $n \to \infty$. (6.5)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1233 **C**

Unique Paper Code : 32357507

Name of the Paper : DSE – 2 Probability Theory and Statistics

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions selecting any **two** parts from each questions no.'s **1** to **6**.
3. Use of scientific calculator is permitted.

1. (i) If the random variable be the time in seconds between incoming telephone calls at a busy switchboard. Suppose that a reasonable probability model for X is given by the probability density function :

$$f_X(x) = \begin{cases} \frac{1}{4}e^{-\frac{x}{4}} & 0 < x < \infty \\ 0 & \text{elsewhere} \end{cases}$$

Show that f_X satisfies the properties of a probability density function. Also show that the probability that the time between successive phone call exceed 4 seconds is 0.3679. (6)

(ii) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} 8x_1x_2 & \text{if } 0 < x_1 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

$$f(x_1, x_2, x_3) = \begin{cases} (x_1 + x_2)e^{-x_3} & \text{if } 0 < x_1 < 1,\ 0 < x_2 < 1,\ 0 < x_3 \\ 0, & \text{elsewhere} \end{cases}$$

Find the regression equation of X_2 on X_1 and X_3. (6.5)

6. (i) Two fair dice are tossed 600 times. Let X denote the number of times a total of 7 occurs. Use Central limit theorem to find $P[95 \le X \le 115]$. (6.5)

(ii) To show how an exponential distribution might arise in practice. If random variable X has an exponential distribution with parameter 0 then find its mean, variance and moment generating function. If X has exponential distribution with mean 2 then find $P[X < 1]$. (6.5)

(iii) If X is a random variable having a binomial distribution with parameter n and θ, then the will finally pass the test on the fourth try?

(6.5)

5. (i) Calculate the correlation coefficient for the following age (in years) of husband's (X) and wife's (Y) : (6.5)

X	23	27	28	28	29	30	31	33	35	36
Y	18	20	22	27	21	29	27	29	28	29

(ii) If X and Y have a bivariate normal distribution, the conditional density of Y given X = x is a normal distribution with the mean,

$$\mu_{Y|x} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu_1)$$

and the variance

$$\sigma^2_{Y|x} = \sigma^2_2\left(1 - \rho^2\right)$$ (6.5)

(iii) The joint density of X_1, X_2 and X_3 is given by

Find $E(X_1X_2^2)$, $E(X_2)$, $E(7X_1X_2^2 + 5X_2)$. (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} e^{-y} & \text{if } 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Find the marginal pdf of X and Y. (6)

2. (i) Let X be a continuous random variable with pdf

$$f(x) = \begin{cases} kxe^{-\lambda x}, & x \ge 0,\ \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

Determine the constant k, mean, variance and the cumulative distribution function of X. (6)

(ii) If a random variable X is uniformly distributed over the interval $[\alpha, \beta]$ then find the mean, variance and moment generating function of X. (6)

(iii) Let the random variables X and Y have the joint pdf

$$f(x, y) = \begin{cases} 6y & \text{if } 0 < y < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the E(Y|x) and E[E(F|x)]. (6)

3. (i) Let (X, y) be a random vector such that the variance off is finite. Then show that Var[E(Y|X)] ≤ Var(Y). (6)

(ii) If X is a binomial variate with parameter n and p then prove that

$$\mu'_{r+1} = \left[np\mu'_r + pq\frac{d\mu'_r}{dp}\right], \text{ where } \mu'_r = E\left[x^r\right] \text{ and r}$$

is a non-negative integer. (6)

(iii) Let the random variables X and Y have the linear conditional means E(Y|x) = 4x + 3 and $E(X|y) = \frac{1}{16}y - 3$. Find the mean of X, mean of Y, the correlation coefficient. (6)

4. (i) Let the random variables X_1 and X_2 have the joint pdf

$$f(x_1, x_2) = \begin{cases} x_1 + x_2 & \text{if } 0 < x_1 < 1,\ 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Show that X_1 and X_2 are not independent. (6.5)

(ii) State and prove the Chebyshev's Theorem. (6.5)

(iii) If the probability is 0.25 that an applicant for driver's license will pass the road test on the given try, what is the probability that an applicant