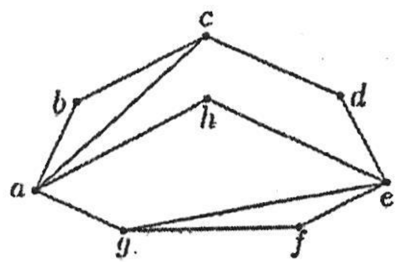
(ii) Define Eulerian circuit. Is the given graph Eulerian? Give reasons for your answer.



(5½)

[This question paper contains 8 printed pages.]

08/12/2022 **Your Roll No..............**

Sr. No. of Question Paper : 1231 **C**

Unique Paper Code : 32357505

Name of the Paper : DSE-2 Discrete Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : V (under CBCS (LOCF) Scheme)

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the given **eight** questions are compulsory to attempt.
3. Do any **two** parts from each of the given **eight** questions.
4. Marks for each part are indicated on the right in brackets.

SECTION I

1. (a) Let N_0 be the set of non-negative integers. Define a relation $\leq$ on N_0 as: For m, n $\in N_0$, $m \leq n$ if m divides n, that is, if there exists $k \leq N_0$: $n = km$. Then show that $\leq$ is an order relation on N_0. (2½)

(b) If '1', '2', '3' denote chains of one, two, three elements respectively and $\overline{3}$ denotes anti chain of three elements, then draw the Hasse diagram for the dual of $L \oplus K$ when $L = \overline{3}$ and $K = 1 \oplus (2 \times 2)$. (2½)

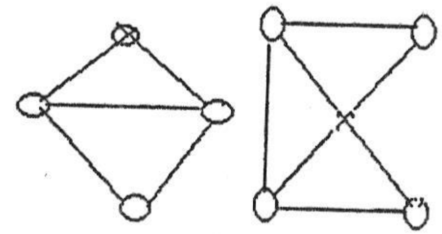
(c) Define maximum and a maximal element of a partially ordered set P. Give an example each for both definitions. (2½)

2. (a) Let P and Q be finite ordered sets and let $\psi: P \to Q$ be a bijective map. Then show that the following are equivalent :

(i) $x < y$ in P iff $\psi(x) < \psi(y)$ in Q

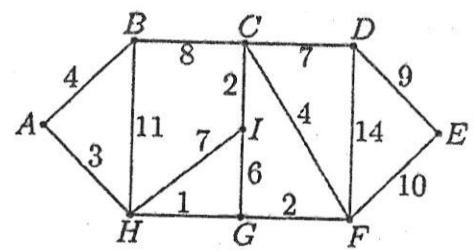
(ii) $x \prec y$ in P iff $\psi(x) \prec \psi(y)$ in Q (3)

(ii) Define isomorphism of graphs. Also label the following graphs so as to show an isomorphism. (5½)



8. (a) Construct a Gray Code of length 3 using the concept of Hamiltonian Cycles. (5½)

(b) Apply Dijkstra's algorithm to find a shortest path from A to all other vertices in the weighted graph shown. (5½)



(c) (i) Does there exist a graph G with 28 edges and 12 vertices each of degree 3 or 6?

SECTION IV

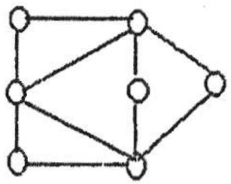
7. (a) (i) Show that the sum of the degrees of the vertices of a pseudograph is an even number equal to twice the number of edges.

(ii) A graph has five vertices of degree 4 and two vertices of degree 2. How many edges does it have? (5½)

(b) (i) Define the degree sequence of a graph. Does there exist a graph with following degree sequence 6, 6, 5, 5, 4, 4, 4, 4, 3?

(ii) Show that the number of vertices of odd in a graph must be even. (5½)

(c) (i) What is a bipartite graph? Determine whether the graph given below is bipartite or not. Give the bipartition sets or explain why the graph is not bipartite.



(b) Define upper bound and lower bound of a subset S of a partially ordered set P. Construct an example of a partially ordered set P and its subset S and give the set of all upper bounds and lower bounds of S. (3)

(c) Let P and Q be ordered sets. Then show that the ordered sets P and Q are order isomorphic iff there exist order preserving maps $\varphi: P \to Q$ and $\psi: Q \to P$ such that :

$\varphi o \psi = id_Q$ and $\psi o \varphi = id_P$ where $id_S : S \to S$ denotes the identity map on S given by: $id_S(x) = x$, $\forall\, x \in S$. (3)

SECTION II

3. (a) Let $D_{60} = \{1, 2, 4, 5, 6, 12, 20, 30, 60\}$ be an ordered subset of $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$, $\mathbb{N}$ being the set of natural numbers. If '$\le$' is defined on D_{60} by $m \le n$ if and only if m divides n then show that D_{60} does not form a lattice. Also Draw the diagram of D_{60} and find elements a, b, c, d $\in D_{60}$ such that $a \vee b$ and $c \wedge d$ do not exist in D_{60}. (5½)

(b) Define sublattice of a lattice. Prove that every chain of a lattice L is a lattice and also a sublattice of L. (5½)

(c) Define modular lattice. Prove that a homomorphic image of modular lattice is modular. (5½)

4. (a) Let L be a lattice. For any a, b, c $\in$ L, show that the following inequalities hold :

(i) $a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(ii) $a \geq c \Rightarrow a \wedge (b \vee c) \geq (a \wedge b) \vee c$

(5)

(b) Let $(L, \wedge, \vee)$ be an algebraic lattice. If we define

$$a \leq b : \Leftrightarrow a \vee b = b$$

then show that $(L, \leq)$ is a lattice ordered set. (5)

(c) Let L_1 and L_2 be distributive lattices. Prove that the product $L_1 \times L_2$ is a distributive lattice. (5)

SECTION III

5. (a) A voting-machine for three voters has YES-NO switches. Current is in the circuit precisely when YES has a majority. Draw the corresponding contact diagram and the switching/circuit diagram. (5½)

(b) Show that a Boolean Algebra is relatively complemented. (5½)

(c) Simplify the polynomial :

$$f = x'yz + x'yz' + x'y'z + xy'z' + xy'z$$

using Quine's McCluskey method. (5½)

6. (a) Define a system of normal forms. Find conjunctive normal form for $p = y'z' + x'yz$. (5)

(b) Simplify the Boolean expression :

$$f = w'xy'z + w'xyz + w'xyz' + wxy'z + wxyz + wxyz' + wx'y'z + wx'yz$$

using Karnaugh Diagram. (5)

(c) Find the symbolic gate representation of the contact diagram : (5)