Name of the course	:	***B.Sc.(H)Mathematics***
Unique Paper Code	:	**32351101**
Name of the Paper	:	**C-1 Calculus (BMATH 101)**
Semester	:	**I**
Duration	:	**3 hours**
Maximum Marks	:	**75**

Attempt any ***four*** *questions. All questions carry equal marks.*

1. (i) Let $f(x)$ be a function defined by $f(x) = x^5 + 5x^4$. Determine the intervals in which this function is increasing or decreasing. Further, determine the points of local maxima and local minima. Find the open intervals in which $f(x)$ is concave up and concave down. Also, determine the point of inflexion, if any.

   (ii) Find the nth derivative of
$$y = e^{3x} \sin x \sin(2x) \sin(3x)$$

   (iii) Find curvature and radius of curvature for
$$\overrightarrow{r(t)} = (e^t \cos t)\hat{\imath} + (e^t \sin t)\hat{\jmath} + 2\hat{k}$$

7.75+6+5

2. Find

   (i) $\lim_{x\to 0}(cosecx - cotx)$

   $\lim_{x\to 0}(\frac{1}{e^x-1} - \frac{1}{x})$

   $\lim_{x\to 1}(1-x)tan\frac{\pi x}{2}$

   (ii) Sketch the ellipse
$$3(x+2)^2 + 4(y+1)^2 = 12$$

   and label the centre, foci, vertices and ends of minor axis.

   (iii) Derive the equation of hyperbola with foci (2,2) and (6,2); asymptotes $y = x - 2$ and $y = 6 - x$. Also, find the centre and vertices of this hyperbola.

(6+6.75+6)

3 (i) Find the volume of the solid that is generated by revolving the region bound by the graphs of $y = x^2$ and $y^2 = x$ about the $y - axis$.

(ii) Use cylindrical shells to find the volume of the solid generated when the region bounded by the curve $y = x^3$, the $x - axis$ and the line $x = 1$ is revolved about the $y - axis$.

(iii)Find the arc length of the curve $f(x) = x^3 + \frac{1}{12x}$ over the interval $[\frac{1}{2}, 2]$. **(7+6.75+5)**

4 (i) Find the tangent vector and parametric equations for the tangent line to the graph of the vector function

$$\overrightarrow{F(t)} = t^{-3}\hat{\imath} + t^{-2}\hat{\jmath} + t^{-1}\hat{k}$$

at the point P corresponding to $t = -1$.

(ii) A particle moves with position vector

$$\overrightarrow{r(t)} = \hat{\imath} + t^2\hat{\jmath} + e^{-t}\hat{k}$$

Find the velocity, speed and acceleration of the particle.

(iii) A projectile is fired from ground level at an angle of 30° with a muzzle speed of 150 m/s. Find the time of flight, the range and the maximum height attained. **(6+6+6.75)**

5 (i) Find all values of k and l such that

$$\lim_{x\to 0} \frac{k+\cos(lx)}{x^2} = -4$$

(ii) Trace the curve $r = 3sin2\theta$

(iii) Trace the conic by removing xy term.

$$6x^2 + 24\,xy - y^2 - 12x + 26y + 11 = 0$$

(6+6+6.75)

6(i) If $y = (sin^{-1}x)^2$, prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$$

(ii) Given $\vec{v}$ and $\vec{a}$ are velocity and acceleration (respectively) of a moving particle at a certain instant of time.

$$\vec{v} = 2\hat{\imath} + 2\,\hat{\jmath} + \hat{k}\ ,\ \ \vec{a} = \hat{\imath} + 2\hat{k}$$

Find tangential and normal components of velocity and acceleration ,unit tangent vector and unit normal vector at this instant.

(iii) Evaluate

$$\int_0^1 x^5 \sqrt{\frac{1+x^2}{1-x^2}}\ dx$$

(6+7.75+5)