19. Find the curvature $\kappa(t)$ for the circular helix

$x = a\cos t,\ y = a\sin t,\ z = ct$, where a and c are constants and $a > 0$.

[This question paper contains 6 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1067 **C**

Unique Paper Code : 32351101

Name of the Paper : BMATH 101 C1 – Calculus

Name of the Course : **CBCS (LOCF) B.Sc. (H) Mathematics**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** the sections are compulsory.
3. **All** questions carry equal marks.
4. Use of non-programmable scientific calculator is allowed.

SECTION – I

Attempt any **four** *questions from Section – I.*

1. Sketch the graph of the function $f(x) = \frac{1}{3}x^3 - 9x + 2$ by finding intervals where it increases and decreases, relative extrema, concavity and inflection points (if any).

2. Evaluate the following limit

$$\lim_{x \to 0} \frac{\tan^{-1}(3x) - 3\tan^{-1} x}{x^3}$$

3. Determine whether the graph of following function has a vertical tangent or a cusp

$$f(x) \text{ — } x^{2/3}(2x + 5)$$

4. It is projected that t years from now, the population of a certain country will be $P(t) = 50e^{0.02t}$ millions.

   (a) At what rate will the population be changing with respect to time 10 years from now?

   (b) At what percentage rate will the population be changing with respect to time t years from now?

SECTION – IV

Attempt any **four** *questions from Section – IV.*

15. For what values of t, the vector function $F(t) = e^t\left[t\hat{i} + \frac{1}{t}\hat{j} + 3\hat{k}\right]$ is continuous?

16. Show that the vector valued function

$$R(t) = \left(2\hat{i} + 2\hat{j} + \hat{k}\right) + \left(\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right)\cos t + \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)\sin t$$

describes the motion of a particle moving in the circle of radius 1, centered at the point (2,2,1) and lying in the plane $x + y - 2z = 2$.

17. A shell fired from ground level at an angle of 45° hits the ground 2000 m away. What is the muzzle speed of the shell?

18. A baseball hit at a 24° angle from 3 ft above the ground just goes over the 9-ft fence 400 ft from home plate. About how fast was the ball traveling, and how long did it take the ball to reach the wall?

5. If $y = e^{m \cos^{-1}x}$, show that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + m^2)y_n = 0.$$

SECTION – II

Attempt any **three** *questions from Section – II.*

6. Sketch the graph of the curve $r = 3 \sin 2\theta$ in polar coordinates.

7. Find an equation for a hyperbola that satisfies the condition that the curve has vertices $(\pm 1, 0)$ and asymptotes $y = \pm 2x$.

8. Describe the graph of the equation :

$$x^2 + 4y^2 + 6x - 40y + 93 = 0.$$

9. Identify and sketch the curve :

$$x^2 + 4xy - 2y^2 - 6 = 0.$$

SECTION – III

Attempt any **four** *questions from Section – III.*

10. Find the arc length of the parametric curve :

$x = \cos t,\ y = t + \sin t$ for $0 \leq t \leq n$.

11. Find the area of the surface generated by revolving the curve $x = \sqrt{25 - y^2},\ -3 \leq y \leq 3$ about the y-axis.

12. The region bounded by the curves $y = x$ and $y = x^2$ is rotated about the line $y = 4$. Compute the volume of the resulting solid.

13. Find the value of the integral $\int_0^{\ln 5} \left(\frac{e^x - e^{-x}}{e^x + e^{-x}} \right) dx$.

14. Evaluate $\int \sin^2 x \cos^4 x \, dx$.