

- (b) Let G be a group such that for any x, y, z in the group, $xy = zx$ implies $y = z$ (called left-right cancellation property). Show that G is Abelian. Give an Example of a non-abelian group in which left-right cancellation property does not hold. (7.5)
- (c) Show that the set $G = \{1, 5, 7, 11\}$ is a group under multiplication modulo 12 with the help of the Cayley table. (7.5)
6. (a) Show that for any integer n , the set $H_n = \{n.x \mid x \in Z\}$ is a subgroup of the group Z of integers under the operation of addition. Further show that $H_2 \cup H_3$ is not a subgroup of Z . (5.5+2)
- (b) Let G be a group. Show that $|aba^{-1}| = |b|$ for all a and b in G ($|x|$ denotes the order of an element x in G). (7.5)
- (c) Show that the group $Z_n = \{0, 1, 2, \dots, n-1\}$ is cyclic under the operation of addition modulo n . How many generators Z_n have? Further, describe all the subgroups of Z_{40} . (2+1+4.5)

(3000)

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 1027

D

Unique Paper Code : 2352011101

Name of the Paper : DSC-1 : Algebra

Name of the Course : **B.Sc. (H) Mathematics,**
UGCF-2022

Semester : I

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

- Write your Roll No. on the top immediately on receipt of this question paper.
 - All questions are compulsory and carry equal marks.
 - Attempt any **two** parts from each question.
- (a) Prove that one root of $x^3 + px^2 + qx + r = 0$ is negative of another root if and only if $r = pq$. (7.5)
 - (b) Solve $x^4 - 2x^3 - 21x^2 + 22x + 40 = 0$, whose roots are in arithmetical progression. (7.5)

P.T.O.

(c) Find all the integral roots of

$$x^7 + 2x^5 + 4x^4 - 8x^2 - 32 = 0. \quad (7.5)$$

2. (a) Find the polar representation of the complex number

$$z = \sin a + i(1 + \cos a), \quad a \in [0, 2\pi) \quad (7.5)$$

(b) Find $|z|$ and $\arg z$ for $z = \frac{(2\sqrt{3} + 2i)^8}{(1-i)^6} + \frac{(1+i)^6}{(2\sqrt{3} - 2i)^8}$

(7.5)

(c) Find the geometric image for the complex number z such that

$$|z + 1 + i| < 3 \text{ and } 0 < \arg z < \frac{\pi}{6} \quad (7.5)$$

3. (a) Let $U_n = \{\epsilon_0, \epsilon_1, \epsilon_2, \dots, \epsilon_{n-1}\}$ be the set of n^{th} roots of unity, where $\epsilon_k = \cos(2k\pi/n) + i \sin(2k\pi/n)$, $k \in \{0, 1, 2, \dots, n-1\}$. Prove the following:

(i) Prove that $\epsilon_j \cdot \epsilon_k \in U_n$ for all $j, k \in \{0, 1, 2, \dots, n-1\}$

(ii) $\epsilon_j^{-1} \in U_n$ for all $j \in \{0, 1, 2, \dots, n-1\}$ (4+3.5)

(b) Let a be an integer, show that there exists an integer k such that

$$a^2 = 3k \text{ or } a^2 = 3k + 1. \quad (7.5)$$

(c) (i) Prove that $\gcd(n, n+1) = 1$ for every natural number n . Find integers x and y such that $n \cdot x + (n+1) \cdot y = 1$.

(ii) Let a, b and c be three natural numbers such that $\gcd(a, c) = 1$ and b divides c . Prove that $\gcd(a, b) = 1$. (4+3.5)

4. (a) Let $n > 1$ be a fixed natural number. Let a, b, c be three integers such that $ac \equiv bc \pmod{n}$ and $\gcd(c, n) = 1$. Prove that $a \equiv b \pmod{n}$.

(7.5)

(b) Solve the congruence, $7x \equiv 8 \pmod{11}$. (7.5)

(c) Solve the following pair of congruences:

$$2x + 3y \equiv 1 \pmod{6}$$

$$x + 3y \equiv 5 \pmod{6} \quad (7.5)$$

5. (a) Let G be the set of all 2×2 real matrices with non-zero determinant. Show that G is a group under the operation of matrix multiplication. Further show that it is not an Abelian Group. (7.5)

P.T.O.