

Name of Course	: CBCS-2 (LOCF) B.Sc. (H) Mathematics
Unique Paper Code	: 32351102
Name of Paper	: C2-Algebra BMATH102
Semester	: I
Duration	: 3 hours
Maximum Marks	: 75 Marks

Attempt any four questions. All questions carry equal marks.

- Solve the equations:
 - $x^4 - 7x^3 + 17x^2 - x - 26 = 0$ given that one root is $3 + 2i$.
 - $2x^3 - x^2 - 22x - 24 = 0$ given that all roots are rational.
 - $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$ given that the product of two of its roots is equal to the other two.
- Find $|z|$, $\arg z$, $\text{Arg } z$, $\arg(-z)$ and $\arg \bar{z}$ for

$$z = (-1 + i)^4 (\sqrt{3} + i)^{10}.$$
 Solve the equation $z^2 + (2i - 3)z + 5 - i = 0$.
- For integers a, b , define $a \sim b$ if and only if $2a + b$ is a multiple of 3. Show that ' \sim ' defines an equivalence relation on \mathbb{Z} . Find the equivalence class of '0' and its quotient set determined by this relation. Evaluate $a + b \pmod{n}$, $ab \pmod{n}$ and $(a - b)^2 \pmod{n}$ for $a = 2003$, $b = -125$ and $n = 37$.
- Show that the following functions $f : A \rightarrow \mathbb{R}$ are one-to-one. Find the range of each function and a suitable inverse:
 - $A = \{x \in \mathbb{R} \mid x \neq 2\}$, $f(x) = 1 - \frac{1}{x-2}$.
 - $A = \{x \in \mathbb{R} \mid x \neq -5\}$, $f(x) = \frac{x-5}{x+5}$.
 Prove that \mathbb{R} , the set of real numbers and the interval $(5,7)$ have the same cardinality.
- Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation which first rotates the given point through an angle of $\pi/6$ about origin and then reflects it about y -axis. Find the Standard matrix of T , denote it by A and check if A is invertible. If yes, find A^{-1} . Determine the eigenspaces of A corresponding to each eigenvalue.
- Check whether the set $\{(1,0,5), (2,1,6), (3,4,0)\}$ is linear independent or not.

Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$. Find the inverse using elementary row operations method using the form $[A : I]$. Also, find the eigenvalues, eigenvectors and eigenspaces of the matrix A . What is the rank of the matrix A ?