[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4530 E

Unique Paper Code : 32351401

Name of the Paper : BMATH408- Partial Differential Equations

Name of the Course : B.Sc.(H) Mathematics

Semester : IV

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All sections are compulsory.
3. Marks of each part are indicated.

Section – I

1. Attempt any two out of the following: [7.5+7.5]

(a) Find the integral surfaces of the equation $u\,u_x + u_y = 1$ for the initial data:

$$x(s,0) = s, y(s,0) = 2s, u(s,0) = s.$$

(b) Apply $\sqrt{u} = v$ and $v(x,y) = f(x) + g(y)$ to solve:

$$x^4\,u_x^2 + y^2\,u_y^2 = 4\,u.$$

(c) Find the solution of the initial–value systems

$$u_t + u\,u_x = e^{-x}v,\ \ v_t - av_x = 0,$$

with $u(x,0) = x$ and $v(x,0) = e^x$.

P.T.O.

Section – II

2. Attempt any one out of the following: [6]

(a) Derive the two- dimensional wave equation of the vibrating membrane

$$u_{tt} = c^2(u_{xx} + u_{yy}) + F,$$

where, $c^2 = T/\rho$, and T is the tensile force per unit length

$F = f/\rho$, and f be the external force, acting on the membrane.

(b) Drive the potential equation $\nabla^2 V = 0$, where ∇^2 is known as Laplace operator.

3. Attempt any two out of the following: [6+6]

(a) Determine the general solution of

$$4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2.$$

(b) Given that the parabolic equation

$$u_{xx} = a\,u_t + b\,u_x + c\,u + f,$$

where the coefficients are constants, by the substitution $u = v\,e^{\frac{1}{2}bx}$ and for the case $c = -(b^2/4)$, show that the given equation is reduced to the heat equation

$$v_{xx} = a\,v_t + g,$$

where $g = f\,e^{-bx/2}$.

(c) Reduce the equation

$$(n-1)^2 u_{rr} - y^{2n} u_{yy} = n\,y^{2n-1} u_y,$$

to canonical form for $n = 1$ and $n = 2$ if possible and also find their solutions.

Section – III

4. Attempt any three parts out of the following: [7+7+7]

(a) Determine the solution of the given below initial-value problem

$$u_{tt} - c^2 u_{xx} = x, \qquad u(x,0) = 0, \quad u_t(x,0) = 3.$$

(b) Obtain the solution of the initial boundary-value problem

$u_{tt} = 9u_{xx}, \quad 0 < x < \infty, \; t > 0,$

$u(x,0) = 0, \quad 0 \leq x < \infty,$

$u_t(x,0) = x^3, \quad 0 \leq x < \infty,$

$u_x(0,t) = 0, \quad t \geq 0.$

(c) Solve:

$u_{tt} = c^2 u_{xx},$

$u(x,t) = f(x) \quad$ on $\quad t = t(x),$

$u(x,t) = g(x) \quad$ on $\quad x + ct = 0,$

where $f(0) = g(0)$.

(d) Determine the solution of the initial boundary-value problem:

$u_{tt} = c^2 u_{xx}, \quad 0 < x < l, \; t > 0$

$u(x,0) = f(x), \quad 0 \leq x \leq l,$

$u_t(x,0) = g(x), \quad 0 \leq x \leq l,$

$u(0,t) = 0, \quad u(l,t) = 0, \quad t \geq 0.$

Section -- IV

5. Attempt any three out of the following: [7+7+7]

(a) Determine the solution of the initial boundary value problem:

$u_t = 4\, u_{xx}, \quad 0 < x < 1, \; t > 0$

$u(x,0) = x^2\,(1-x), \quad 0 \leq x \leq 1,$

$u(0,t) = 0, \quad u(l,t) = 0, \quad t \geq 0.$

(b) Determine the solution of the initial boundary value problem by the method of separation of variables:

$u_{tt} = c^2 u_{xx}, \quad 0 < x < \pi, \; t > 0$

$u(x,0) = 0, \quad 0 \leq x \leq \pi,$

$u_t(x,0) = 8\sin^2 x, \quad 0 \leq x \leq \pi,$

$u(0,t) = 0, \quad u(\pi,t) = 0, \quad t \geq 0.$

(c) Solve by using method of separation of variables:

$u_{tt} - u_{xx} = h,$ $0 < x < 1,\ t > 0,$ h is a constant

$u(x,0) = x^2,$ $0 \le x \le 1,$

$u_t(x,0) = 0,$ $0 \le x \le 1,$

$u(0,t) = 0, \quad u(1,t) = 0,$ $t \ge 0.$

(d) State and prove the uniqueness of solution of the heat conduction problem.