(c) Apply mid-point method (second order Runge-Kutta method) to solve the initial value problem using h=0.2 on the interval [0,0.4] :

$$\frac{dy}{dx} = -2xy^2,\ y(0) = 1. \qquad (6.5)$$

[This question paper contains 8 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : **6613** **E**

Unique Paper Code : 32355402_OC

Name of the Paper : GE-4 Numerical Methods

Name of the Course : **CBCS (Other than B.Sc. (H) Mathematics)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. All the **six** questions are compulsory.

3. Attempt any **two** parts from each question.

4. Use of non-programmable scientific calculator is allowed.

1. (a) Define significance digits and truncation error with examples. If an approximate value of π is given by 3.14285 and its true value is 3.14159 then find the absolute and relative errors. (6)

(b) Find the root correct to three decimal places by using Newton-Raphson method for the equation

$$f(x) = x^3 - 7x^2 + 16x - 12 = 0$$

with initial approximation $x_0 = 1$. (6)

(c) Perform four iterations of bisection method to obtain the root of the equation :

$$f(x) = 4x^2 - 3 = 0$$

in the interval [0,1]. (6)

2. (a) Perform three iterations of Secant method to obtain the root of the equation :

(c) Use Euler's method to find y(0.4) from the following differential equation

$$\frac{dy}{dx} = xy,\ y(0) = 1,\ h = 0.1.$$ (6)

6. (a) Apply Gaussian Quadrature three-point formula to approximate the value of the integral :

$$I = \int_0^1 \frac{dx}{1+x^2}$$ (6.5)

(b) Solve the following initial value problem using Modified Euler's method, with h = 0.1, x ∈ [1,12]. Compare with the exact solution.

$$\frac{dy}{dx} = x + y,\ y(0) = 1,\ h = 0.1$$ (6.5)

5. (a) The following data represents the function $f(x) = e^x$

x	0.0	0.3	0.6	0.9	1.2
f(x)	1.0000	1.8221	3.3201	6.0496	11.0232

Find $f'(1.2)$, $f''(1.2)$ using the Newton's Backward difference method. Compute the magnitude of the error. (6)

(b) Find the approximate value of the integral

$$I = \int_1^6 \frac{dx}{(1+x)^2},$$ using

(i) Trapezoidal rule.

(ii) Simpson's $\frac{1}{3}$ rule.

Also calculate the error in each case. (6)

$$f(x) = \cos x - xe^x = 0$$

with initial approximation $x_0 = -1$ and $x_1 = 1$. (6.5)

(b) Define rate of convergence. Determine the rate of convergence of the secant method. (6.5)

(c) Perform three iterations of Newton method to solve the system of equations

$$x^2 + xy + y^2 = 7$$

$$x^3 + y^3 = 9$$

with initial approximation $x_0 = 1.5$ and $y_0 = 0.5$. (6.5)

3. (a) Solve the linear system Ax = b using Gauss Elimination method with pivoting where (6)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

(b) Solve the following tri-diagonal system Ax = b using the Gauss Thomas method.

$$A=\begin{bmatrix}2&2&0&0\\2&4&4&0\\0&1&3&3\\0&0&2&5\end{bmatrix},\quad b=\begin{bmatrix}4\\6\\7\\10\end{bmatrix}$$ (6)

(c) Perform three iterations of the Gauss-Jacobi method starting with X0 = (0,0,0) to solve the following system of equations: (6)

$$4x + y + z = 2$$

$$x + 5y + 2z = -6$$

$$x + 2y + 3z = -4$$

4. (a) Find the Lagrange's interpolating polynomial passing through the points

x	−1	1	4	7
f(x)	−2	0	63	342

Hence, interpolate at x = 5. (6.5)

(b) Prove the following :

(i) $(1+\Delta)(1-\nabla) = 1$

(ii) $\delta = \nabla(1-\nabla)^{-1/2}$ (6.5)

(c) Generate the difference table for the data

x	−2	−1	0	1	2	3
f(x)	15	5	1	3	11	25

Obtain Gregory Newton forward difference interpolating polynomial and hence interpolate the value of f(0.5). (6.5)