Find y(0.5) and y(0.75) by using the modified Euler's method. Also find the absolute error at each step given, that the exact solution of the IVP is $y=\sqrt{2e^x-1}$. (6.5)

1/6/23 (Eve)

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 6207 E

Unique Paper Code : 32355402

Name of the Paper : GE-4: Numerical Methods

Name of the Course : **CBCS / LOCF (Other than B.Sc. (H) Mathematics Hons.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. Use of scientific calculator is allowed.

1. (a) Round-off the number 34.64867 correct up to three significant digits and then calculate absolute percentage error. (6)

(b) Find the absolute, relative and percentage error if the approximate value is 2.7182 and the true value is 2.71828182. (6)

(c) Perform four iterations of the bisection method to find the approximate root of the equation $x^5 + 2x - 1 = 0$ in interval (0,1). (6)

2. (a) By using the Regula–Falsi method find the approximate root, correct up to two decimal places, of the equation $x^3 - 6x + 4 = 0$ in the interval (0,1). (6.5)

(b) By performing three iterations of the secant method find the approximate root of the equation $x^3 - 5 \sin x + 1 = 0$ in the interval (0,1). (6.5)

(c) Using the Newton-Raphson method find the approximate value of the cube root of 25. Perform three iterations of the method by taking initial approximation $x_0 = 2.8$. (6.5)

6. (a) Approximate the value of $(\ln 2)^{\frac{1}{3}}$ from $\int_0^1 \frac{x^2}{1+x^3}\,dx$ using Trapezoidal rule and Simpson's $\frac{1}{3}$ rule. (6.5)

(b) Apply the Fleun method to approximate the solution of the initial value problem

$$\frac{dy}{dx} = \frac{1}{2}(1+x)y^2, \quad 0 \le x \le 1, \quad y(0) = 1,$$

by using 5 steps. (6.5)

(c) Given the initial value problem (IVP):

$$\frac{dy}{dx} = \frac{e^x}{y}, \quad y(0) = 1.$$

x	0	1	2	3	4
f(x)	8	4	6	20	52

(6)

(b) For the function f(x) = ln x, approximate f′(2) by Richardson extrapolation using central difference formula $f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$ with h = 0.1 and h = 0.05. (6)

(c) Use the formula

$$f'(x_i) \approx \frac{3f(x_i)-4f(x_i-h)+f(x_i-2h)}{2h}$$

to approximate the derivative of f(x) = sin x at $x_i = \pi$, taking h = 1, 0.1, 0.01. (6)

3. (a) Using Gauss elimination method solve the following system of linear equations : (6)

$$x + 4y - z = -5$$

$$x + y - 6z = -12$$

$$3x - y - z = 4.$$

(b) Show that $E - 1 = \frac{1}{2}\delta^2 + \delta\mu$. (Note: Symbols have their own meaning) (6)

(c) Find the Lagrange interpolating polynomial which fits into the given data and approximate the value of f(5.5).

x	5	6	9
f(x)	12	13	14

(6)

4. (a) By using the initial solution (0,0,0), perform three iterations of the Gauss Seidel method for the following system of linear equations : (6.5)

$$10x + 2y + z = 9$$

$$2x + 20y - 2z = -44$$

$$-2x + 3y + 10z = 22.$$

(b) Obtain the piecewise linear interpolating polynomial for the function f(x) defined by the given data and by using it estimate the value of f(3).

x	1	2	4	8
f(x)	3	7	21	73

(6.5)

(c) Following table gives the amount of half yearly premium for policies maturing at different ages: (6.5)

Age (in years)	45	50	55	60	65
Premium (in Rs.)	114.84	96.16	83.32	74.48	68.48

Make the difference table. Obtain the forward Gregory-Newton interpolating polynomial and estimate the premium for policy maturing at the age of 46.

5. (a) For the following data, find f′(2) and f″(2) by using forward difference formulae

$$f'(x_i) \approx \frac{f(x_i + h) - f(x_i)}{h} \text{ and}$$

$$f''(x_i) \approx \frac{f(x_i) - 2f(x_i + h) + f(x_i + 2h)}{h^2}$$