

6207

8

Find $y(0.5)$ and $y(0.75)$ by using the modified Euler's method. Also find the absolute error at each step given, that the exact solution of the

IVP is $y = \sqrt{2e^x - 1}$. (6.5)

(1000)

1/6/23 (EVE)

[This question paper contains 8 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 6207 E

Unique Paper Code : 32355402

Name of the Paper : GE-4: Numerical Methods

Name of the Course : CBCS / LOCF (Other than B.Sc. (H) Mathematics Hons.)

Semester : IV

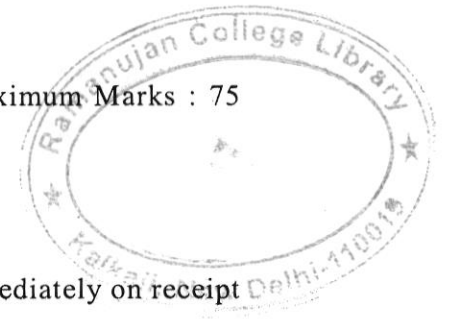
Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
 2. Attempt any **two** parts from each question.
 3. Use of scientific calculator is allowed.
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1. (a) Round-off the number 34.64867 correct up to three significant digits and then calculate absolute percentage error. (6)

P.T.O.



- (b) Find the absolute, relative and percentage error if the approximate value is 2.7182 and the true value is 2.71828182. (6)
- (c) Perform four iterations of the bisection method to find the approximate root of the equation $x^5 + 2x - 1 = 0$ in interval (0,1). (6)
2. (a) By using the Regula-Falsi method find the approximate root, correct up to two decimal places, of the equation $x^3 - 6x + 4 = 0$ in the interval (0,1). (6.5)
- (b) By performing three iterations of the secant method find the approximate root of the equation $x^3 - 5 \sin x + 1 = 0$ in the interval (0,1). (6.5)
- (c) Using the Newton-Raphson method find the approximate value of the cube root of 25. Perform three iterations of the method by taking initial approximation $x_0 = 2.8$. (6.5)

6. (a) Approximate the value of $(\ln 2)^{\frac{1}{3}}$ from $\int_0^1 \frac{x^2}{1+x^3} dx$ using Trapezoidal rule and Simpson's $\frac{1}{3}$ rule. (6.5)
- (b) Apply the Fleun method to approximate the solution of the initial value problem
- $$\frac{dy}{dx} = \frac{1}{2}(1+x)y^2, \quad 0 \leq x \leq 1, \quad y(0)=1,$$
- by using 5 steps. (6.5)
- (c) Given the initial value problem (IVP):

$$\frac{dy}{dx} = \frac{e^x}{y}, \quad y(0)=1.$$

x	0	1	2	3	4
f(x)	8	4	6	20	52

(6)

(b) For the function $f(x) = \ln x$, approximate $f'(2)$ by Richardson extrapolation using central difference

formula $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$ with $h = 0.1$ and

$h = 0.05$.

(6)

(c) Use the formula

$$f'(x_i) \approx \frac{3f(x_i) - 4f(x_i - h) + f(x_i - 2h)}{2h}$$

to approximate the derivative of $f(x) = \sin x$ at $x_i = \pi$, taking $h = 1, 0.1, 0.01$.

(6)

3. (a) Using Gauss elimination method solve the following system of linear equations : (6)

$$x + 4y - z = -5$$

$$x + y - 6z = -12$$

$$3x - y - z = 4.$$

(b) Show that $E - 1 = \frac{1}{2}\delta^2 + \delta\mu$. (Note: Symbols have their own meaning) (6)

(c) Find the Lagrange interpolating polynomial which fits into the given data and approximate the value of $f(5.5)$.

x	5	6	9
f(x)	12	13	14

(6)

4. (a) By using the initial solution (0,0,0), perform three iterations of the Gauss Seidel method for the following system of linear equations: (6.5)

$$10x + 2y + z = 9$$

$$2x + 20y - 2z = -44$$

$$-2x + 3y + 10z = 22.$$

- (b) Obtain the piecewise linear interpolating polynomial for the function $f(x)$ defined by the given data and by using it estimate the value of $f(3)$.

x	1	2	4	8
f(x)	3	7	21	73

(6.5)

- (c) Following table gives the amount of half yearly premium for policies maturing at different ages: (6.5)

Age (in years)	45	50	55	60	65
Premium (in Rs.)	114.84	96.16	83.32	74.48	68.48

Make the difference table. Obtain the forward Gregory-Newton interpolating polynomial and estimate the premium for policy maturing at the age of 46.

5. (a) For the following data, find $f'(2)$ and $f''(2)$ by using forward difference formulae

$$f'(x_i) \approx \frac{f(x_i + h) - f(x_i)}{h} \text{ and}$$

$$f''(x_i) \approx \frac{f(x_i) - 2f(x_i + h) + f(x_i + 2h)}{h^2}$$