Subjects / Teachers	A	B	C	D
I	28	47	36	38
II	36	43	43	46
III	43	38	36	33
IV	47	48	31	38
V	48	38	41	43
VI	43	52	43	49

(c) Define rectangular fair Game. Using Maxmin and Minmax Principle, find the maximum pay-off for player 1 will have and minimum pay-off for player 2 for the following pay-off matrix :

$$\text{Player 2}\overset{\text{Player 1}}{\begin{bmatrix} 10 & 8 & 4 \\ 9 & -5 & 15 \\ -1 & 7 & 6 \end{bmatrix}}.$$

(d) Convert the following Game Problem into a Linear Programming Problem for player A and player B and solve it by Simplex Method :

$$\text{Player A}\overset{\text{Player B}}{\begin{bmatrix} 6 & 3 & 5 \\ 2 & 5 & 1 \end{bmatrix}}.$$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4874 **E**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF)– B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt any **two** parts from each question.

3. **All** questions carry equal marks.

1. (a) Solve the following Linear Programming Problem by Graphical Method :

Maximize $2x + y$

subject to $x + 2y \le 10$

$x + y \le 6$

$x - y \le 2$

$x - 2y \le 1$

$x \ge 0,\ y \ge 0.$

(b) Define a Convex Set. Show that the set S defined as :

$S = \{(x, y) \mid y^2 \ge 4ax;\ x \ge 0,\ y \ge 0\}$ is a Convex Set.

(c) Find all basic feasible solutions of the equations :

$x_1 + 2x_2 + 4x_3 + x_4 = 7$

$2x_1 - x_2 + 3x_3 - 2x_4 = 4$

(d) Prove that to every extreme point of the feasible region, there corresponds a basic feasible solution of the Linear Programming Problem :

Minimize $z = cx$

subject to $Ax = b,\ x \ge 0$

2. (a) Let us consider the following Linear Programming Problem :

Source \ Destination	A	B	C	D	E	Supply
I	10	10	11	12	10	20
II	13	14	11	15	10	35
III	11	10	17	12	15	40
Demand	25	10	30	15	15	

5. (a) Solve the following cost minimization Transportation Problem :

Origin \ Destinations	I	II	III	IV	Availability
A	14	11	13	12	22
B	13	17	10	15	15
C	13	15	16	14	8
Requirements	7	12	17	9	

(b) A University wish to allocate four subjects and six teachers claim that they have the required knowledge to teach all the subjects. Each subject can be assigned to one and only one teacher. The cost of Assignment of subject to each teacher is given in table below. Allocate the subjects to appropriate faculty members for optimal Assignment. Also find two Teachers who are not assigned any course.

Maximize $3x_1 + 4x_2 - 3x_3$

Subject to $x_1 - 2x_2 + 5x_3 \geq 2$

$3x_1 + 7x_2 - 4x_3 = -8$

$x_1 \leq 0,\ x_2 \geq 0,\ x_3$ is unrestricted.

(b) Prove that if the Primal Problem has a finite optimal solution then the Dual also has a finite optimal solution and the two optimal objective function values are equal.

(c) Using Complementary Slackness Theorem, find optimal solutions of the following Linear Programming Problem and its Dual.

Minimize $2x_1 + 15x_2 + 5x_3 + 6x_4$

subject to $x_1 + 6x_2 + 3x_3 + x_4 \geq 2$

$-2x_1 + 5x_2 - x_3 + 3x_4 \leq -3$

$x_1, x_2, x_3, x_4 \geq 0.$

(d) For the following cost minimization Transportation Problem find initial basic feasible solutions by using North West Comer rule, Least Cost Method and Vogel's Approximation Method. Compare the three solutions (in terms of the cost) :

Minimize $z = cx$

subject to $Ax = b$, $x \geq 0$

Let $(x_B, 0)$ be a basic feasible solution corresponding to a basis B where $x_B = B^{-1}b$. Suppose $z_0 = c_B x_B$ is the value of objective function such that $z_j - c_j \leq 0$ for every column a_j in A. Show that z_0 is the minimum value of z of the problem and that the given basic feasible solution is optimal feasible solution.

(b) Let $x_1 = 1,\ x_2 = 1,\ x_3 = 1$ be a feasible solution to the system of equations :

$$x_1 + x_2 + 2x_3 = 4$$

$$2x_1 - x_2 + x_3 = 2$$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Using Simplex Method, find the solution of the following Linear Programming Problem :

Maximize $5x_1 + 4x_2$

subject to $x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 4$

$5x_1 + x_2 \leq 15$

$x_1, x_2 \geq 0.$

(d) Solve the following Linear Programming Problem by Big - M Method :

Maximize $-x_1 - x_2 + x_3$

subject to $x_1 - x_2 - x_3 = 1$

$-x_1 + x_2 + 2x_3 \geq 1$

$x_1, x_2, x_3 \geq 0.$

3. (a) Solve the following Linear Programming Problem by Two Phase Method :

Maximize $-x_1 + 3x_2$

subject to $x_1 + x_2 \geq 1$

$-2x_1 + 3x_2 \leq 6$

$x_1 \geq 0, x_2 \geq 0.$

(b) Find the solution of given system of equations using Simplex Method :

$$5x_1 + 2x_2 = 14$$

$$2x_1 + x_2 = 6$$

Also find the inverse of A where $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

(c) Find the solution of the following Linear Programming Problem :

Minimize $3x_1 + 2x_2$

subject to $-x_1 + x_2 \leq 1$

$5x_1 + 3x_2 \leq 15$

$x_1 \geq 0, x_2 \geq 3/2.$

(d) Find the optimal solution of the Assignment Problem with the following cost matrix :

Job \ Machines	I	II	III	IV	V	VI
A	3	4	6	5	4	9
B	5	4	9	7	6	10
C	8	7	6	5	4	6
D	7	4	5	11	10	4
E	5	6	7	8	5	9
F	4	3	5	6	7	4

4. (a) Find the dual of the following Linear Programming Problem :