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- (d) Let f be defined on  $\mathbb{R}$  and suppose that  $|f(x) - f(y)| \le (x - y)^2$  for all  $x, y \in \mathbb{R}$ . Prove that f is a constant function. (5)
- 5. (a) Let f be differentiable function on an open interval (a, b). Then show that f is increasing on (a, b) if f'(x) ≥ 0. (5)
  - (b) If  $y = e^{\tan^{-1}x}$ , prove that (5)

$$(1 + x^2)y_{n+2} + (2(n+1)x - 1)y_{n+1} + n(n+1)y_n = 0.$$

- (c) If  $y = \cos(m \sin^{-1} x)$ , find  $y_n(0)$ . (5)
- (d) Stating Taylor's theorem find Taylor series expansion of e<sup>x</sup>.
   (5)

6. (a) Find 
$$\lim_{x \to +\infty} \left[ x - \ln \left( x^2 + 1 \right) \right].$$
 (5)

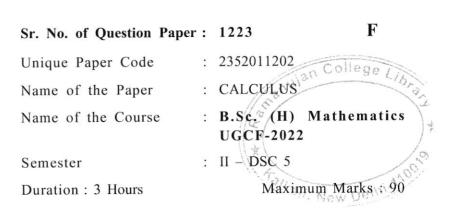
- (b) Determine the position and nature of the double points on the curve (5)  $x^{3} - y^{2} - 7x^{2} + 4y + 15x - 13 = 0.$
- (c) Sketch a graph of the rational function showing the horizontal, vertical and oblique asymptote (if

any) of 
$$y = \frac{x^2 - 2}{x}$$
. (5)

(d) Sketch the curve in polar coordinates of  $r = \sin 2\theta$ . (5)

(1000)

[This question paper contains 4 printed pages.]



Your Roll No.....

## **Instructions for Candidates**

- 1. Write your Roll No. on the top immediately on receipt of this question paper.
- 2. Attempt **all** questions by selecting **three** parts from each question.
- 3. All questions carry equal marks.
- 4. Use of Calculator is not allowed.
- (a) State and prove the sequential criterion for the limit of a real valued function. (5)
  - (b) Use  $\in -\delta$  definition of limit to establish the following limit: (5)

$$\lim_{x \to 2} \frac{1}{1 - x} = -1.$$

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(c) Let  $f: \mathbb{R} \to \mathbb{R}$  be defined as (5)

 $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$ 

Show that f has a limit only at x = 0.

- (d) Let  $A \subseteq \mathbb{R}$ , let  $f: A \to \mathbb{R}$ , and let  $c \in \mathbb{R}$  be a cluster point of A. If  $\lim_{x \to c} f > 0$ , then show that f(x) > 0 for all  $x \in A \cap V_{\delta}(c), x \neq c$ . (5)
- (a) If f is continuous at x<sub>0</sub> and g is continuous at f(x<sub>0</sub>) then prove that the composite function g ° f is continuous at x<sub>0</sub>.

(b) Let 
$$f(x) = \frac{1}{x} \sin \frac{1}{x^2}$$
 for  $x \neq 0$  and  $f(0) = 0$ . Show

that f is discontinuous at 0. (5)

- (c) State Intermediate Value Theorem. Prove that xe<sup>x</sup> = 1 for some x in (0,1).
- (d) Let f be a continuous real-valued function with domain (a, b). Show that if f(r) = 0 for each rational number r in (a, b), then f(x) = 0 for all x ∈ (a, b).
- 3. (a) Prove that if a real valued function f is continuous on [a, b] then it is uniformly continuous on [a, b].(5)

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- (b) Show that the function  $f(x) = \frac{1}{x}$  is uniformly
  - continuous on  $(a, \infty)$  for a > 0 but it is not uniformly continuous on (0,1). (5)
- (c) Let f(x) = |x| + |x 1|,  $x \in \mathbb{R}$ . Draw the graph and give the set of points where it is not differentiable. Justify also. (5)
- (d) Prove that if f and g are differentiable on R, if f(0) = g(0) and if f'(x) ≤ g'(x) for all x ∈ R, then f(x) ≤ g(x) for x ≥ 0.
- 4. (a) State and prove Mean Value Theorem. (5)
  - (b) State Intermediate Value Theorem for derivatives. Suppose f is differentiable on ℝ and f(0) = 0, f(1) = 1, f(2) = 1.

(i) Show that  $f'(x) = \frac{1}{2}$  for some  $x \in (0,2)$ .

(ii) Show that 
$$f'(x) = \frac{1}{7}$$
 for some  $x \in (0,2)$ .

(c) Prove that  $(\sin x - \sin y) \le |x - y|$  for all x, y  $\in \mathbb{R}$ . (5)

P.T.O.