some vectors $v_1, v_2, \ldots, v_n \in V$, then $\{v_1, v_2, \ldots, v_n\}$ is linearly independent set in V. Is the converse true? Justify with examples. Under what condition the converse holds true, Justify.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2033 **F**

Unique Paper Code : 2354001202

Name of the Paper : Introduction to Linear Algebra

Name of the Course : **GE**

Semester : II

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** question by selecting **two** parts from each question.
3. Part of the questions to be attempted together.
4. **All** questions carry equal marks.

1. (a) If x and y are vectors in $\mathbb{R}^n$ then prove that

$$|x.y| \leq (\|x\|)(\|y\|).$$

Also verify it for vectors $x = [-1, -1, 0]$ and

$y = [\sqrt{2}, \sqrt{2}, \sqrt{2}]$.

(b) Use Gaussian Elimination to solve the following system of linear equations. Indicate whether the system is consistent or inconsistent. Give the complete solution set, if consistent.

$$3x - 3y - 2z = 23$$

$$-6x + 4y + 3z = -40$$

$$-2x + y + z = -12$$

(c) Use Gauss-Jordan row reduction method to find the complete solution set for the following system of equations.

$$4x - 8y - 2z = 0$$

$$3x - 5y - 2z = 0$$

$$2x - 8y + z = 0$$

6. (a) Let $L: R^3 \to R^3$ be given by

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 5 & 1 & -1 \\ -3 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

(i) Is $[1, -2, 3]$ in Ker(L)? Why or why not?

(ii) Is $[2, -1, 4]$ in Range(L)? Why or why not?

(b) Consider the linear Transformation $L: M_{3\times3} \to M_{3\times3}$ given by

$$L(A) = A - A^T,$$

where, A^T represents the transpose of the matrix A. Find the Ker(L) and Range(L) of the Linear Transformation, and verify

$$\dim(\mathrm{Ker}(L)) + \dim(\mathrm{Range}(L)) = \dim(M_{3\times3}).$$

(c) Suppose that $L: V \to W$ is a linear Transformation. Show that if $\{L(v_1), L(v_2), ..., L(v_n)\}$ is a linearly independent set of n distinct vectors in W, for

(b) Consider the linear Transformation L: $P_3(R) \to M_{2\times2}(R)$ given by

$$L(ax^3 + bx^2 + cx + d) \begin{bmatrix} -3a-2c & -b+4d \\ 4b-c+3d & -6a-b+2d \end{bmatrix}.$$

Find the matrix for L with respect to the standard basis for P_3 and $M_{2\times2}$. Also, Find the dimension of Ker(L) and Range(L).

(c) Let L: $P_3 \to P_4$ be given by

$$L(p) = \int p, \text{ for, } p \in P_3.$$

Where, $\int p$ represents the integration of p. Find the matrix for L with respect to the standard basis for P_3 and P_4. Use this matrix to calculate $L(4x^3 - 5x^2 + 6x - 7)$ by matrix multiplication.

2. (a) Define rank of a matrix. Find Rank of the matrix

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 4 & 2 & 3 \\ -1 & 2 & 4 \end{bmatrix}.$$

(b) Find all Eigen values corresponding to the matrix A. Also, find the eigenspace for each of the eigen value of the matrix A, where $A = \begin{bmatrix} 12 & -51 \\ 2 & -11 \end{bmatrix}$.

(c) Let $A = \begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ -2 & 4 & -3 \end{bmatrix}$. Determine whether the vector $X = [5, 17, -20]$ is in row space of the matrix A. If so, then express X as a linear combination of the rows of A.

3. (a) Let V be a vector space. Let H and K be subspaces of V. Prove that $H \cap K$ is also a subspace of V. Give an example to show that $H \cup K$ need not be a subspace.

(b) Prove or disprove that the set S ={ [1,2,1], [1,0,2], [1,1,0]} forms a basis of R^3.

(c) Use the Diagonalization Method to determine whether the matrix A is diagonalizable. If so, specify the matrices D and P and verify that $P^{-1}AP = D$, where

$$A = \begin{bmatrix} 19 & -48 \\ 8 & -21 \end{bmatrix}.$$

4. (a) (i) Let S = {(x,y,z): x + y = z ∀x, y, z ∈ R}. Show that S is a subspace of R^3.

(ii) Consider the vector space $M_{3\times3}(R)$ and the sets

T_1: the set of nonsingular 3×3 matrices

T_2: the set of singular 3×3 matrices.

Determine T_1 and T_2 are subspaces of $M_{3\times3}(R)$ or not?

(b) Define linearly independent subset of a finite dimensional vector space V. Use Independent Test Method to find whether the set S = {[1,0,1,2], [0,1,1,2], [1,1,1,3], [−1,2,3,1] } of vectors in R^4 is linearly independent or linearly dependent.

(c) Define basis of a vector space V. Show that the subset {[1,0,−1], [1,1,1],[1,2,4]} of R^3 forms a basis of R^3.

5. (a) (i) Determine whether the following function is a Linear Transformation or not?

$L: R^3 \to R^3$ given by

$L([x_1, x_2, x_3]) = [x_2, x_3, x_1]$.

(ii) Suppose $L: R^3 \to R^3$ is a linear operator and

L[1,0,0] = [−2,1,0], L[0,1,0] = [3,−2,1] and L[0,0,1] = [0,−1,3].

Find L[−3,2,4]. Give a formula for L[x, y, z], for any [x, y, z] ∈ R^3.