(b) Write the dual of the linear programming problem given in part (A) above and solve it.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 3833 **E**

Unique Paper Code : 12271202

Name of the Paper : Mathematical Methods for Economics – II

Name of the Course : **B.A. (Hons.) Economics**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. There are **4** questions in all.
3. **All** questions are compulsory.
4. All parts of a question must be answered together.
5. Use of simple calculator is allowed.

1. Attempt any **four** of the following : (4×5.5=22)

(a) Examine the definiteness of the following quadratic forms :

(i) $f(x, y) = -x^2 - xy - y^2$

(ii) $-x^2 + xy - y^2$ subject to $5x - 2y = 5$

(b) Find the domain of the following functions and draw the sets in the xy plane :

(i) $f(x,y) = \sqrt{x+1} + \sqrt{y}$

(ii) $g(x,y) = \sqrt{9-(x^2+y^2)}$

[In lieu of B for PWD]

Find the domain of the following functions :

(b) Find the area of the region bounded by $y = x^3$, the x-axis and the line $x = 4$ for $x > 0$.

(c) The marginal cost function of firm is $MC = (\log x)^2$. Find the total cost of 100 units if the cost of producing one unit is Rs. 22.

(d) Find the differential equation of the family of circles passing through the origin and having center on the x-axis.

4. Attempt any **one** of the following : (1×7=7)

(a) Solve the following linear programming problem

Max $3x + 4y$

subject to $\begin{cases} 3x + 2y \le 6 \\ x + 4y \le 4 \end{cases}$ $x \ge 0$ $y \ge 0$

Compute the increase in the criterion function if the first constraint changes to $3x + 2y \le 8$.

(ii) $f(x, y) = x - y - x^2$

(e) Maximize $U(x, y) = 100 - e^{-x} - e^{-y}$ subject to $px + qy = m$.

(i) Solve for x and y as functions of p, q, and m.

(ii) Prove that x and y are homogeneous of degree 0 as functions of p, q, and m.

3. Attempt any **three** of the following : (3×6=18)

(a) Show that any function $x = x(t)$ that satisfies the equation $(t - a)^2 + x^2 = a^2$ is a solution of the following differential equation :

$$2tx\frac{dx}{dt} + t^2 - x^2 = 0$$

(i) $f(x,y) = \sqrt{x+1} + \sqrt{y}$

(ii) $g(x,y) = \sqrt{9-(x^2+y^2)}$

(iii) $f(x,y) = \ln(1-x^2-y^2)$

(c) Show that $x^2 + y^2 = 6$ is a level curve of $f(x,y) = \sqrt{x^2+y^2} - x^2 - y^2 + 2$ and that all the level curves of f must be circles centered at the origin.

(d) Let $z = \frac{1}{2}\ln(x^2 + y^2)$. Find : $\frac{d^2z}{dx^2} + \frac{d^2z}{dy^2}$.

(e) Examine the homogeneity of the following functions :

(i) $F(x_1, x_2, x_3) = \frac{(x_1x_2x_3)^2}{x_1^4 + x_2^4 + x_3^4}$

(ii) $G(x_1, x_2, x_3) = \left(ax_1^e + bx_2^e + cx_3^e\right)$

2. Attempt any **four** of the following : (4×7=28)

(a) Suppose a monopolist is practicing price discrimination in the sale of a product by charging different prices in two separate markets. Suppose the demand curves are $P_1 = 100 - Q_1$ and $P_2 = 80 - Q_2$ and suppose the cost function is $6(Q_1 + Q_2)$. How much should be sold in the two markets to maximize profits? What are the prices charged? How much profit is lost if price discrimination is made illegal?

(b) Use the extreme value theorem and find the extreme points and extreme values for f(x, y) defined over S when

$$f(x, y) = x^2 + y^2 + y - 1, \quad S = \{(x,y): x^2 + y^2 \le 1\}$$

(c) Let the function g be given by

$$g(x, y) = x^4 y^4 + 2x^2 y^2 - 2x^2 - 2y^2$$

Find the stationary points of g and classify them as local maxima, local minima, global maxima, or global minima.

(d) Examine the concavity/convexity of the following functions :

(i) $f(x, y, z) = (x + 2y + 3z)^2$