[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1058 **D**

Unique Paper Code : 2342011103

Name of the Paper : Mathematics for Computing

Name of the Course : **B.Sc. (H) Computer Science**

Semester : I

Duration : 3 Hours Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. The paper has **two** sections. **Section A** is compulsory. Each question is of **5** marks.

3. Attempt any **four** questions from **Section B**. Each question is of **15** marks.

Section A

1. (a) Write the following system of equations in matrix form. Reduce the augmented matrix into row echelon form. (5)

$$x_1 + 3x_2 + x_3 = 1$$

$$-4x_1 - 9x_2 + 2x_3 = -1$$

$$-3x_1 - 6x_3 = -3$$

(b) Define a convex set. Show if $C = \{x_2 : 2x_1 + 3x_2 = 7\} \subset R^2$ is a convex set. (5)

(c) Show that the transformation defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$ is not linear. (5)

(d) Find the characteristic polynomial of the following matrix (5)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix}$$

(e) Let $a = -2\hat{i} + 3\hat{j} + 5\hat{k}$ and $b = \hat{i} + 2\hat{j} + 3\hat{k}$ be two vectors. Find the value of the dot product of these two vectors. (5)

(f) Determine whether or not the vectors (4, 1, –2), (–3, 0, 1) and (1, –2, 1) form a basis of R^3. (5)

Section B

2. (a) For what values of λ and μ do the following system of equations is consistent. (7)

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

(b) Find the inverse of the following matrix using Gauss Jordan method. (8)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

3. (a) Determine whether the system has a nonzero solution. (7)

$$x + 2y - 3z = 0$$

$$2x + 5y + 2z = 0$$

$$3x - y - 4z = 0$$

(b) Apply Gram Schmidt orthonormalization process to obtain an orthonormal basis for the subspace of R^4 generated by the vectors. (1, 1, 0, 1), (1, –2, 0, 0), (1, 0, –1, –2). (8)

4. (a) Use the Cayley-Hamilton theorem to find

$(A - 2I)(A - 3I)$ where $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$. (7)

(b) What is a subspace? Let Y be the set of vectors in R^4 of the form [a, 0, b, 0]. Prove that Y is a subspace of R^4. (8)

5. (a) Calculate the curl and divergence for the following vector field. (7)

$$\vec{F} = x^3y^2\hat{i} + x^2y^3z^4\hat{j} + x^2z^2\hat{k}$$

(b) What is a positive definite matrix? Is the following matrix positive definite? (8)

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

6. (a) Let $a = [1, 1, 0]$, $b = [3, 2, 1]$ and $c = [1, 0, 2]$, Find the angle between: a, b and b, c. (3)

(b) If $\phi(x,y,z) = 3x^2y - y^3z^2$, find $\nabla\phi(\text{grad}\phi)$ at the point $(1, -2, -1)$. (4)

(c) Diagonalize the matrix (8)

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 4 & -1 \\ -2 & -4 & 4 \end{bmatrix}$$

7. (a) If V is an inner product space, then show that $\langle v, au + bw \rangle = a \langle v, u \rangle + b \langle v, w \rangle$ where a and b are scalars and v, u, w are vectors in V. (7)

(b) Suppose that three banks in a certain town are competing for investors. Currently, Bank A has 40% of the investors, Bank B has 10%, and Bank C has the remaining 50%. Suppose the towns folk are tempted by various promotional campaigns to switch banks. Records show that each year Bank A keeps half of its investors, with the remainder switching equally to Banks B and C. However, Bank B keeps two-thirds of its investors, with the remainder switching equally to Banks A and C. Finally, Bank C keeps half of its investors, with the remainder switching equally to Banks A and B. Find the distribution of investors after two years. (8)