(d) Show that $f(x) = \frac{1}{x}$ is not uniformly continuous on $[0,1]$.

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 5025 **E**

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) Let S be a non empty bounded set in $\mathbb{R}$. Let $a > 0$, and let $aS = \{as: s \in S\}$. Prove that $\inf aS = a \inf S$, $\sup aS = a \sup S$.

(b) Define order completeness property of real numbers.

(c) Define limit point of a set. Show that the set $\mathbb{N}$ of natural numbers has no limit point.

(d) State and prove Archimedean property of real numbers.

2. (a) Show that the function defined as

$$f(x)=\begin{cases} x, & \text{if } x \text{ is irrational} \\ -x, & \text{if } x \text{ is rational} \end{cases}$$

is continuous only at $x = 0$.

(b) Show that the function f defined by $f(x) = x^2$ is uniformly continuous on $[-2,2]$.

6. (a) Define Riemann integrability of a bounded function f on a bounded closed interval [a, b]. Show that the function f defined on [a, b] as

$$f(x)=\begin{cases} 0 & \text{when } x \text{ is rational} \\ 1 & \text{when } x \text{ is irrational} \end{cases}$$

is not Riemann integrable.

(b) Test for convergence the series

$$\sum_{n=1}^{\infty}(-1)^n \frac{\cos n\alpha}{\sqrt{n^3}}, \ \alpha \text{ being real.}$$

(c) State D'Alembert's ratio test for the convergence of a positive term series. Use it to test for convergence the series $\sum_{n=1}^{\infty}\frac{n!}{n^n}$.

5\. (a) State Leibnitz test for convergence of an alternating series of real numbers. Apply it to test for convergence the series

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

(b) Show the sequence defined by $\langle a_n \rangle = \langle n^2 \rangle$ is not a Cauchy sequence.

(c) Prove that the sequence $\langle a_n \rangle$ defined by the relation,

$$a_n = 1,\ a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots\cdots\cdots + \frac{1}{(n-1)!},\quad (n \geq 2),$$

converges.

(d) Prove that every continuous function is integrable.

(c) Define an open set. Prove that every open interval is an open set. Which of the following sets are open.

(i) $]2, \infty[$

(ii) $[3,4[$

(d) Let A and B be bounded nonempty subsets of R, and let $A + B = \{a + b : a \in A, b \in B\}$. Prove that $\sup (A + B) = \sup A + \sup B$.

3\. (a) Prove that every convergent sequence is bounded. Justify by an example that the converse is not true.

(b) Prove that the sequence $\langle a_n \rangle$ defined by the recursion formula :

$$a_1 = \sqrt{7},\ a_{n+1} = \sqrt{7 + a_n}$$

converges to the positive root of $x^2 - x - 7 = 0$.

(c) State Cauchy's convergence criterion for sequences. Check whether the sequence $\langle a_n \rangle$, where

$$a_n = 1 + \frac{1}{5} + \frac{1}{9} + \cdots \cdots\cdots + \frac{1}{4n-3}$$

is convergent or not.

(d) Test for convergence the series :

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots\cdots\cdots$$

4. (a) Prove that, if the series $\sum u_n$ converges, then $\lim_{n\to\infty} u_n = 0$. Show by an example that the converse is not true.

(b) Test for convergence the series :

$$\sum_{n=1}^{\infty} \frac{2.4.6 \ldots\ldots\ldots (2n+2)}{3.5.7 \ldots\ldots\ldots (2n+3)} x^{n-1} \qquad (x > 0)$$

(c) Let $\langle a_n \rangle$ be a sequence defined by :

$$a_1 = 1,\ a_{n+1} = \frac{3 + 2a_n}{2 + a_n},\ n \geq 1.$$

Show that $\langle a_n \rangle$ is convergent and find its limit.

(d) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.