Unique Paper Code	:	32377905
Name of the Paper	:	Time Series Analysis (DSE-1(i))
Name of the Course	:	B.Sc. (Hons.) Statistics under CBCS
Semester	:	V
Duration	:	3 hours
Maximum Marks	:	75 Marks

Instruction for Candidates

Attempt any *four* questions. All questions carry equal marks. Use of a calculator is allowed.

1. Let y_t, $t = 1,2, \ldots n$, be the given time series. Derive the logistic curve

$$y_t = \frac{k}{1 + e^{a+bt}}$$

where k, a, and b are constants such that $b < 0$. Measurements of the mean height (in cms) attained by sunflower plants, at interval of seven days was recorded as under:

Days	7	14	21	28	35	42	49	56	63	70	77	84
Mean Height (in cms)	17.93	36.36	67.76	98.1	131	169.5	205.5	228.3	247.1	250.5	253.8	254.5

It is known that a logistic curve is the best curve to represent trend for the above time series data. If the average maximum height attained by the variety of sunflower under study is 260 cm and the harvesting of the crop is done between 110 to 120 days, then use the most suitable method for fitting this curve. Also, calculate the fitted trend values. What will be the average height attained on days 105 and 112?

2. In the usual notations, prove that

$$\frac{1}{m_1 m_2 \ldots m_r}[m_1][m_2] \ldots [m_r]y_0 = y_0 + \frac{m_1^2 + m_2^2 + \cdots + m_r^2 - r}{24}\delta^2 y_0$$

where $\frac{1}{m}[m]$ stands for the simple arithmetic mean of 'm' terms. Hence or otherwise find weights associated with [2k+1,2]. If k=5, show that the weights are symmetric about the origin, sum to unity and are independent of the part of series under consideration.

3. Explain Ratio-to-Trend method for measuring the seasonal variations in a time series mentioning one merit and demerit each. The number of traffic accidents in Calcutta in four quarters of a year during the period 1977-79 are given below. Fit a straight-line trend to the annual averages and use the same to obtain the trend values for each quarter. Assuming a multiplicative model, use Ratio-to-Trend method to calculate indices of seasonal variations for each quarter within a year:

Year	Q I	Q II	Q III	Q IV
1977	165	135	140	180
1978	152	121	127	163
1979	140	100	105	158

4. Distinguish between a strict stationary process and a weak stationary process. Define autocorrelation function for a stationary process. Do the autocorrelation functions uniquely determine a moving average process? Justify your answer with the help of a

suitable example. Find MA processes which have for autocorrelation $\rho_0 = 1,\ \rho_1 = 0.4,\ \rho_k = 0; k \geq 2$. For each of the four MA processes

$$y_t = 1.6z_t - 0.8z_{t-1} + 0.4z_{t-2} - 0.2z_{t-3}$$
$$y_t = -0.2z_t + 0.4z_{t-1} - 0.8z_{t-2} + 1.6z_{t-3}$$
$$y_t = 0.4z_t - 0.2z_{t-1} + 1.6z_{t-2} - 0.8z_{t-3}$$
$$y_t = -0.8z_t + 1.6z_{t-1} - 0.2z_{t-2} + 0.4z_{t-3}$$

show that all have the first three autocorrelation coefficients equal to $-\frac{42}{85}, \frac{4}{17}, -\frac{8}{85}$ and the rest are zero.

5. Enumerate the steps involved in Box-Jenkins approach to forecasting. For the model
$$(1-B)(1-0.2B)y_t = (1-0.5B)z_t$$
where $\{Z_t\}$ is a discrete-time, purely random process such that $E(z_t) = 0, Var(z_t) = \sigma_z^2$ and successive values of z_t are independent, find the forecasts for one- and two-steps-ahead. Hence or otherwise, show that a recursive expression for forecasts three or more steps ahead is given by
$$\hat{y}_N(h) = 1.2\hat{y}_N(h-1) - 0.2\hat{y}_N(h-2)$$
Obtain the variance of the one- and two-steps-ahead forecast errors. Further, if $z_N = 1$, $y_N = 4$, $y_{N-1} = 3$, $\sigma_z^2 = 2$, find $\hat{y}_N(2)$ and the standard error of the corresponding forecast error.

6. Establish that for a steady model the algorithm obtained using Brown's discounted regression is the same as simple exponential smoothing. Use simple exponential smoothing method to find the forecasts for the following sales data, taking an initial forecast as 25 and smoothing coefficient 0.4

Days	1	2	3	4	5	6	7	8
Sales	26	28	23	27	24	30	26	27