Unique Paper Code: 32371501
Name of the Course: B.Sc. (Hons.) STATISTICS: CBCS
Name of the Paper: Stochastic Processes and Queuing Theory
Semester: V
Duration: 3 hours
Maximum Marks: 75

Instructions for Candidates:
Attempt ***four*** questions in all. All questions carry equal marks. Use of calculator is allowed.

1. Let N be a random variable denoting the total number of eggs of an insect and g_n be the probability that an insect lays n eggs. Each egg has probability p of surviving. Let X be a random variable denoting the event of survival of an egg and $\{p_k\}$ denotes its probability distribution. The random variables N and X are independent. Obtain the probability generating function of total number of surviving eggs. If N follows Poisson distribution with mean rate 6 per insect and X_i is a Bernoulli variate with probability ¾, then find the mean and variance of total number of surviving eggs.

2. Derive the differential difference equations for the linear growth process with emigration. Hence obtain the mean population size and also the second order differential equation of mean population size assuming that population starts with two ancestors.

3. Let the number of persons taking a particular vaccine for Flu follow Poisson Process with parameter λ per week. A vaccinated person has probability p of not developing serious pneumonia (independent of others who received the vaccine). Derive the probability distribution of number of individuals who can get Pneumonia even after taking the vaccine. What will be the average number of persons affected with Pneumonia in 3 weeks duration if λ =8 and p =.9?

4. Consider the classical gambler's ruin problem. Assume that the two players A and B play with an initial capital of i and N-i.
   i) Find the probability that gambler A ends up with total money involved in the game.
   ii) Show that the probability that either A or B will end up with all the money.
   iii) What will be the impact on probability of ruin of player A if the stakes are reduced to half ?
   iv) If i = \$ 10, N = \$100, p = .25 then what is the expected Gain involved in the game.

5. In a city there are three types of grocery stores (I, II and III).It has been observed that there is always a shift of customers from one store to another. A study was made on January 1, 2021 and it was found that 1500 people shopped at store I whereas 1400 and 2000 shopped at stores II and III respectively. Each month store I losses 10 % of its customers to store II and 15% to store III. Store II loses 10 % each to stores I and III. Store III loses 50% of its customers to store I and 10% to store II.
   i) What will be the number of customers who shop at Store I, II and III next month?
   ii) What proportion of customers will each store retain by March 1, 2021?
   iii) Assuming the same patterns continues, what will be the long run distribution of customers among the three stores? Hence write the limiting distribution P^n as n tends to infinity.
   iv) Which Store will be most popular in the long run?

6. Consider a self-service store with one cashier, FCFS queue discipline and infinite capacity. Assume Poisson arrivals and Exponential service times. Identify a suitable queueing model and obtain the probability of n customers in steady state under this model. Suppose that 8 customers arrive on an average every 5 minutes and the cashier can serve 10 in 5 minutes. Find (i) the average number of customers in the queue and (ii) probability that the number of customers in the system is more than 4.