Unique Code: 32371301
Name of Course: B.Sc. (H) Statistics Under CBCS
Name of Paper: Sampling Distributions
Semester: III
Duration : 3 hours
Maximum marks : 75

Instructions for candidates
Attempt four questions in all. All questions carry equal marks.

1. A discrete random variable is specified by f (-9)= f(a) =1/8, f(0)= ¾. Compute P ($|X| \geq 2\sigma$) and compare it with Chebychev's inequality bound. Comment on your result. Consider another variate X_k with the following distribution,

$$P[X_n = \pm 1] = \frac{1}{2}(1 - 2^{-n})$$

$$P[X_n = \pm 2^{-n}] = 2^{-n-1}$$

Examine if Weak Law of Large Numbers holds for the independent sequence { X_k}?

2. Given the following value of the F distribution, $F_{4,8}$ (0.05) = 3.84, find $F_{8,4}$ (0.95) and state the result used to find this value. Show both the points graphically clearly marking the corresponding areas. If X ~ $F_{(m,n)}$, then obtain the distribution of U =$\frac{mX}{n+mX}$.

3. Let $X_{(1)}$, $X_{(2)}$,….. $X_{(n)}$, be a random sample from uniform (0,1) population. Find the co efficient of correlation between $X_{(1)}$ and $X_{(n)}$.

4. Show that chi square variate with zero degrees of freedom is a degenerate variate. Let $X_1, X_2, \ldots, X_n$ are independent variables each distributed as $N(\mu, \sigma^2)$. Find

$$V_{ar}[T] \text{ and } r(\bar{X}, T), \text{ where } T = \frac{\bar{X} - \mu}{S/\sqrt{n}}, \quad \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \text{ and } S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}.$$

5. Compare and analyze the graphs of $\chi 2$, t and F distributions. Explain with an example in each case how you will determine the critical region and significant values in each case for the purpose of hypothesis testing using these distributions any chosen level of significance. Let X_i ~ N (i , i^2) i = 1, 2, 3 be independent variates. Using X_1, X_2 and X_3 give an example of a statistic that has the following distributions,

(a) t distribution with 2 degrees of freedom
(b) F (1,2)

6. How would you decide about the minimum sample size required in sampling from large populations to achieve a desired level of precision? A random sample of 500 is drawn from a large number of freshly minted coins. The mean weight of the coins in the sample is 30 grams and the standard deviation is 1.25 grams. What are the limits which have a 49 to 1 chance of including the mean weight of all the coins? How large a sample would have to be drawn to make

these limits differ by only 0.1 grams assuming that the standard deviation of the whole distribution is 1.25 grams?